Mathematics 172

Quiz 33 Name: key

You must show your work to get full credit.

1. The follow system gives the population sizes of two competing species.

$$\frac{dx}{dt} = .15x\left(\frac{300 - x - .7y}{300}\right)$$
$$\frac{dy}{dt} = .08y\left(\frac{150 - 3x - y}{150}\right)$$

(a) What are values of the following:

The carrying capacity of the x-species 300

The carrying capacity of the y-species 150

The intrinsic growth rate of the x species .15

The intrinsic growth rate of the y species .08

(b) If $x(5) = 150$ and $y(5) = 70$ compute the following

$x'(5) =$ 7.575 $\qquad$ $y'(5) =$ −13.813

$x'(5) = .15x\left(\frac{300 - x - .7y}{300}\right)$

$= .15(150)\left(\frac{300 - 150 - .7(70)}{300}\right)$

$= 7.575$

$y' = .08(70)\left(\frac{150 - 3(150) - 70}{150}\right)$

$= -13.813$

(c) Estimate the following

$x(5.5) \approx$ 153.79 $\qquad$ $y'(5.5) \approx$ 63.09

$x(5.5) \approx x(5) + x'(5)(.5)$

$= 150 + 7.575(.5)$

$= 153.79$

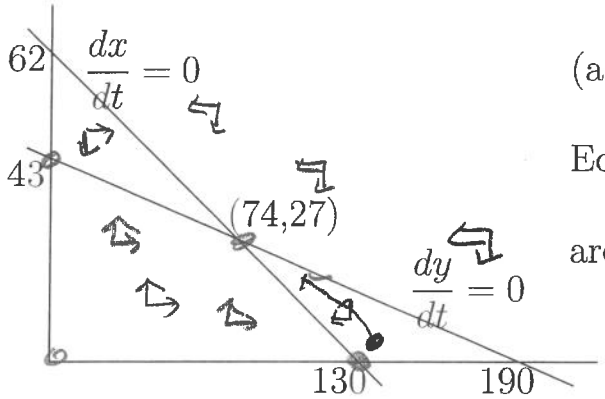
$y(5.5) \approx y(5) + y'(5)(.5)$

$= 70 + (-13.813)(.5)$

$= 63.09$

2. Consider populations of competing species modeled by the equations

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x - \alpha y}{K_1} \right)$$
$$\frac{dy}{dt} = r_2 y \left(\frac{K_2 - \beta x - y}{K_2} \right)$$



(a) If the phase space is as on the right:

(i) What are the equilibrium points?

Equilibrium points are (0,0), (130,0), (0,43), (74,27)

(ii) Draw in the arrows showing how points are moving.

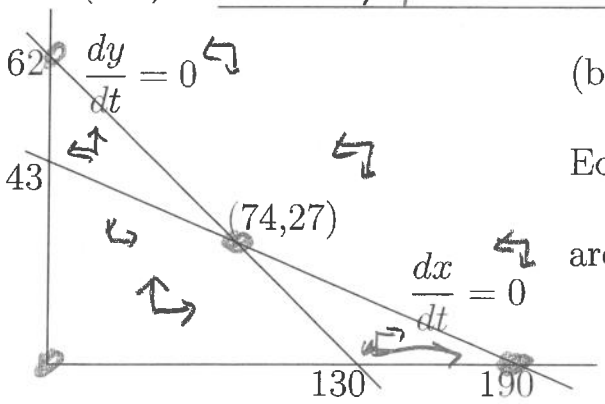
(iii) What are the stable equilibrium points?

Stable points are: (74,27)

(iv) Circle one: ***Competitive coexistence*** ***Competitive exclusion***

(v) If $x(0) = 178$ and $y(0) = 6$ estimate the following

$x(132) \approx$ 74 $y(132) \approx$ 27



(b) If the phase space is as on the right:

(i) What are the equilibrium points?

Equilibrium points are (0,0), (190,0), (0,62), (74,27)

(ii) Draw in the arrows showing how points are moving.

(iii) What are the stable equilibrium points?

Stable points are: (190,0), (0,62)

(iv) Circle one: ***Competitive coexistence*** ***Competitive exclusion***

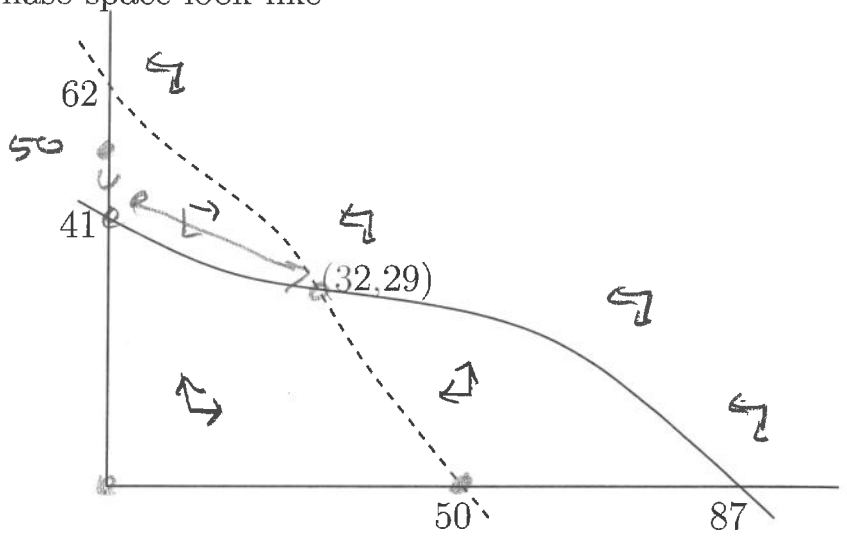
(v) If $x(0) = 178$ and $y(0) = 6$ estimate the following

$x(132) \approx$ 190 $y(132) \approx$ 0

3. Let the population sizes of two competing species be modeled by the equations

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y)$$

and let the phase space look like



where the dotted curve (------) is where $f(x,y) = 0$ and the solid curve (____)

(a) What are the equilibrium points?

Equilibrium points are: (0,0), (50,0), (0,41), (32,29)

(b) Draw in arrows show the direction that points are moving.

(c) What are the stable equilibrium points?

Stable points are (32,29)

(d) What is the carrying capacity of the x species?

$K_1 =$ 50

(e) Circle one: ***Competitive coexistence*** ***Competitive exclusion***

(f) If $x(0) = 5$ and $y(0) = 50$ estimate the following:

$x(100) \approx$ 32 $y(100) \approx$ 29

(g) If $x(0) = 0$ and $y(0) = 50$ estimate the following:

$x(100) \approx$ 0 $y(100) \approx$ 41

4. For the Predator–Victim system

$$\frac{dV}{dt} = .3V - .03VP = V(.3 - .03P) \quad \hat{P} = \frac{.3}{.03} = 10$$

$$\frac{dP}{dt} = -.9P + .01VP = P(-.9 + .01V) \quad \hat{V} = \frac{.9}{.01} = 90$$

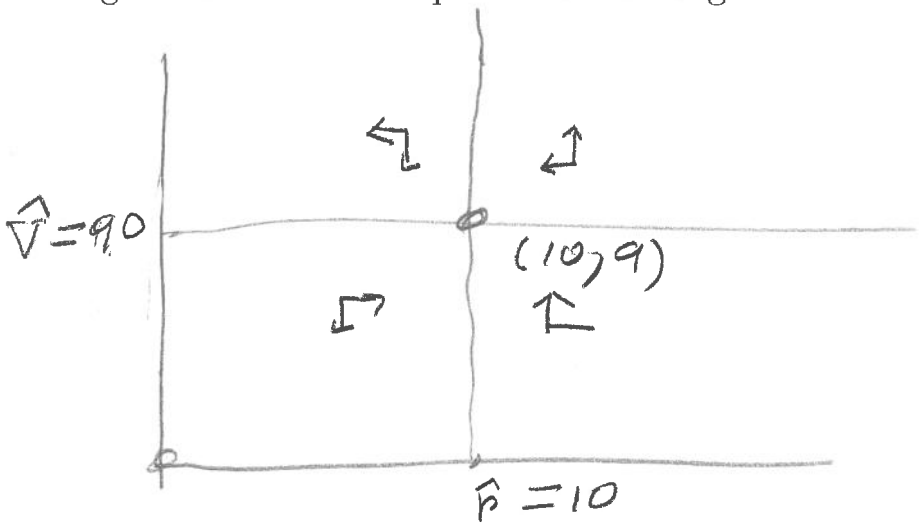
(a) What is the intrinsic growth rate of the victims?

Growth rate is .3

(b) What is the death rate of the predators when there is no prey?

The rate is −.9

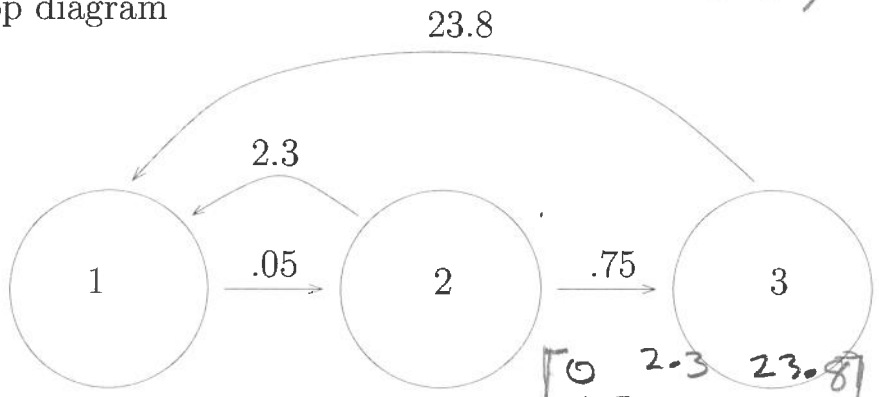
(c) Draw the phase space labeling the lines where $\frac{dV}{dt} = 0$ and $\frac{dP}{dt} = 0$ and put in arrows showing the direction that points are moving.



(d) What are the equilibrium points?

The equilibrium points are (10,0), (10,90)

5. For the loop diagram



(a) What is the Leslie matrix? $L =$ $\begin{bmatrix} 0 & 2.3 & 23.8 \\ .05 & 0 & 0 \\ 0 & .75 & 0 \end{bmatrix}$

(b) What does the number .75 mean?

The proportion of stage 2 individuals that survive to stage 3

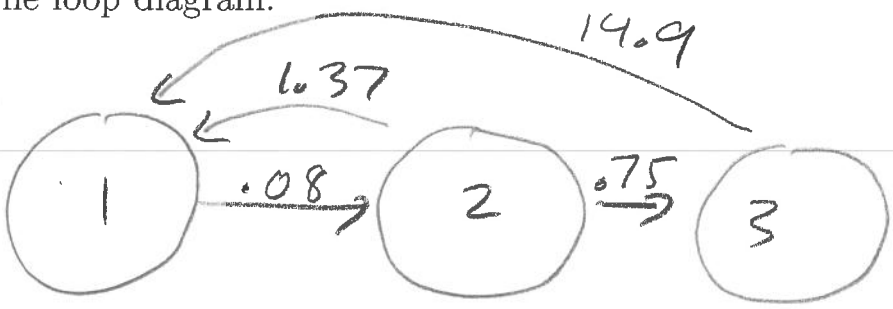
(c) What does the number 23.8 mean?

The average number of offspring that live to stage 1 that turn into a stage 3 individual

6. For the Leslie matrix

$$L = \begin{bmatrix} 0 & 1.37 & 14.9 \\ 0.08 & 0 & 0 \\ 0 & 0.75 & 0 \end{bmatrix}$$

(a) Draw the loop diagram:



(b) If $\vec{N}(0) = \begin{bmatrix} 410 \\ 27 \\ 20 \end{bmatrix}$ Find the following

$\vec{N}(1) = \begin{bmatrix} 334.94 \\ 32.8 \\ 20.25 \end{bmatrix}$

$\vec{N}(20) = \begin{bmatrix} 374.63 \\ 28.37 \\ 23.00 \end{bmatrix}$

$\vec{N}(50) = \begin{bmatrix} 387.63 \\ 30.57 \\ 23.00 \end{bmatrix}$

(c) What is the proportion in each stage for $\vec{N}(50)$?

Total = 387.63 + 30.57 + 23.00 = 441.25

Proportion in stage 1 $\frac{387.63}{441.25} = .878$

Proportion in stage 2 $\frac{30.57}{441.25} = .069$

Proportion in stage 3 $\frac{23.00}{441.25} = .052$