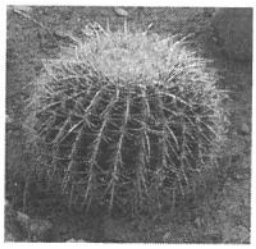
Mathematics 172

Quiz 4 Name: Key

***You must show your work to get full credit.***



Assume that a type of barrel cactus has a crushing weight of 101 psi. (A psi is $1\text{lb}/\text{in}^2$.) If a cactus which is 10 inches tall weighs 30 pounds and has a base of area 50 $\text{in}^2$, what is the critical height where a barrel cactus will crush itself?

Here are some steps to solve this. Let $C_\lambda$ be a version of the cactus described above by a scale factor of $\lambda$.

**1.** What is the weight of $C_\lambda$? Weight scales by $\lambda^3$ $30\lambda^3$ lb

Weight$(C_\lambda) = 30\lambda^3$ lb

**2.** What is a area of the base of $C_\lambda$? Area scales by $\lambda^2$. $50\lambda^2$ in

Area base$(C_\lambda) = 50\lambda^2$ in

**3.** What is a average pressure of the of the cactus on its base? (This is the total weight divided by the area of the base.)

$$\text{Average pressure}(C_\lambda) = \frac{30\lambda^3 \text{ lb}}{50\lambda^2 \text{ in}^2} = .6\lambda \text{ lb/in}^2 = .6\lambda \text{ psi}$$

**4.** What value of $\lambda$ makes the average pressure at the base equal to 101psi.

Solve $.6\lambda = 101$

$\lambda = \frac{101}{.6} = 168.3$ $\lambda =$ 168.3

**5.** What is the tallest this type of cactus can get before it crushes under its own weight?

Height scales by $\lambda$ so

Height $= \lambda(10 \text{ in}) = 168.3$ in Height = 1683 in