Mathematics 172

Quiz 5 **Name:** Key

You must show your work to get full credit.

1. Compute the following derivatives.

(a) $y = 3e^{2x} - 5x^4$ $y' =$ $6e^{2x} - 20x^3$

$y' = 3 \cdot 2e^{2x} - 5(4)x^3 = 6e^{2x} - 20x^3$

(b) $P(t) = P_0 e^{rt}$ where r and P_0 are constants.

(use $\frac{d}{dt} e^{rt} = re^{rt}$) $\frac{dP}{dt} =$ $rP_0 e^{rt}$

2. Let r be a constant and let $f(t)$ satisfy

$$f'(t) = rf(t).$$

Let

$$y = e^{-rt} f(t).$$

(a) Use the product rule to find the derivative of y.

$y' = (e^{-rt})' f(t) + e^{-rt} f'(t)$ $y' =$ $-re^{-rt} f(t) + e^{-rt} f'(t)$

$= -re^{-rt} f(t) + e^{-rt} f'(t)$

(b) Use your formula from part (a) and that $f'(t) = rf(t)$ to show $y' = 0$.

$y' = -re^{-rt} f(t) + e^{-rt}(rf(t))$

$= -re^{-rt} f(t) + re^{-rt} f(t)$

$= 0$

(c) We know that a function that has zero derivative is constant, thus $y = C$ for some constant C. That is $e^{-rt} f(t) = C$. Solve this for $f(t)$.

$e^{-rt} f(t) = C$ $f(t) =$ Ce^{rt}

$f(t) = \frac{C}{e^{-rt}} = Ce^{rt}$