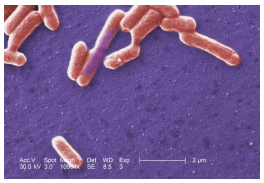
Mathematics 172

Quiz 9 **Name:** Key

You must show your work to get full credit.



Part of the point of this quiz is to show how fast exponential functions grow. Under ideal conditions the bacterium E. coli will double every 1/3 hours. A single E. coli weights 10^{-15}kg.

1. We start with a colony of a single E. coli. Assuming that the weight of the colony doubles every 1/3 hours, give a formula for the weight, $W(t)$, of the colony after t hours.

The basic formula for unconstrained growth is $W(t) = W_0 e^{rt} = 10^{-15} e^{rt}$. To find r set

$$W\left(\tfrac{1}{3}\right) = 10^{-15} e^{\frac{r}{3}} = 2\,(10^{-15})$$

$$e^{\frac{r}{3}} = 2$$

$$\frac{r}{3} = \ln(2)$$

$$r = 3\ln(2) = 2.07944$$

$W(t) = 10^{-15} e^{2.07944t}$

2. The weight of the Earth is 6.0×10^{24}kg. How long does it take our colony of E. coli to have the same weight as the Earth?

Solve

$$W(t) = 10^{-15} e^{2.07944t} = 6 \times 10^{24}$$

$$e^{2.07944t} = \frac{6 \times 10^{24}}{10^{-15}} = 6 \times 10^{39}$$

$$2.07944t = \ln(6 \times 10^{39})$$

$$t = \frac{\ln(6 \times 10^{39})}{2.07944} = 44.047 \text{ hours}$$

So it takes less than 2 days.

Time to reach mass of Earth is 44.047 hrs.