Mathematics 172

This is a solution to the problem on Quiz 12 where I messed up by mixing up the units.

1. The crushing pressure of red ceder is 4,560 psi. Assume that a red ceder with a height of 5 feet = 60 inches, the area of its base is .4 feet^2 = 57.4 in. and weighs 60 lbs. Then what is the critical height where a red ceder crushes itself under its own weight?

Solution: Let λ be a scaling factor and consider a red ceder that is scaled be a factor of λ.

- The scaled height is 60λ in.
- The scaled base area is $57.4\lambda^2$ in^2.
- The scaled weight is $60\lambda^3$ lb.

Therefore the pressure on the base is

$$\frac{\text{weight}}{\text{area of base}} = \frac{60\lambda^3 \text{lb}}{57.4\lambda^3 \text{in}^2} = 1.045\lambda \text{lb/in}^2.$$

So at the critical size, where the pressure on the base is equal to the crushing pressure is when

$$1.045\lambda \text{lb/in}^2 = 4{,}560 \text{ lb/in}^2.$$

Solving for λ gives

$$\lambda = \frac{4{,}560}{1.045} = 4363.6$$

Therefore the critical height where crushing occurs is

$$60\lambda = 60(4363.6) = 261{,}818.2\text{in} = 21{,}818.2\text{ft} = 4.1322\text{miles}.$$

Clearly this is not the main constraint on the height of the tree.