Mathematics 172 Test 1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (5 points) A rocket accelerates at a rate of $a = 45$ ft/sec^2. Use that 1 mile $=$ 5,280 feet and 1 hour $=$ 3,600 sec to express a is miles/hours^2.

$a =$ 110,454.5 $\frac{\text{mile}}{\text{hr2}}$

$$a = 45 \frac{\text{ft}}{\text{sec2}} = 45 \frac{\frac{\text{miles}}{5280}}{\left(\frac{\text{hr}}{3600}\right)^2}$$

$$= \frac{45\,(3600)^2}{5280} \frac{\text{mile}}{\text{hr2}}$$

$$= 110954.5 \frac{\text{miles}}{\text{hr2}}$$

2. (15 points) The crushing pressure of a type of cottonwood is 400 psi (lbs/in^2). If a 10 foot (= 120 in) tall cottonwood weights 100 lbs and the area of its base is 60 in^2. How tall can a cottonwood get before it is crushed by its own weight.

Critical height is 28,794 in = 2,399.5 ft

Scale the tree by a factor of λ.

scaled height $= 120\lambda$ in

scaled base area $= 60\lambda^2$ in2

scaled weight $= 100\lambda^3$ lb.

The scaled pressure at base is

$$\frac{\text{weight}}{\text{Area}} = \frac{100\lambda^3 \text{ lb}}{60\lambda^2 \text{ in2}} = 1.667\lambda \text{ lb/in2}$$

The critical scaling factor is when this ~~equals~~ equals the crushing pressure i.e

$$1.667\lambda = 400$$

$$\text{so} \quad \lambda = \frac{400}{1.667} = 239.95$$

Thus the critical height is

$$120\lambda = 120\,(239.95) = 28794 \text{ in}$$
$$= 2399.5 \text{ ft}$$

3. (20 points) The zoo sets up a large tank for fish from Lake Tanganyika. They add some plants to the tank and the plants have 60 grams of brown algae on them. After 4 weeks there is 114 grams of the algae in the tank. Assume the algae grows with a constant intrinsic growth rate r for the first several months.

(a) What is the intrinsic growth rate of the algae? Include units in your answer.

If it grows at constant intrinsic growth rate we have

$r =$ $r = .160$ grams/wk.

$P(t) = P_0 e^{rt} = 60\, e^{rt}$

$P(4) = 60 e^{r(4)} = 114$

$e^{4r} = 114/60$

$4r = \ln(114/60)$

$r = \ln(114/60)/4$

$= .160$

(b) If $P(t)$ is the number of grams of brown algae in the tank t weeks after the plants are added, then give a formula for $P(t)$.

$P(t) =$ $60\, e^{.16 t}$

(c) How long until there is 500 grams of brown algae in the tank?

Time to 500 grams is. 13.25 weeks

Solve $60 e^{.16 t} = 500$

$e^{.16t} = 500/60$

$.16 t = \ln(500/60)$

$t = \ln(500/60)/.16 = 13.25$

4. (15 points) A typical male gorilla has a body length to 67 in and weighs 374 lbs. Its mandible (jaw bone) is 4.5 in.

Gigantopithecus is the largest primate in the fossil record. It lived in southern China from roughly 2 million to 350,000 years ago. It is believed to have been related to the orangutans, but is generally restored is having the same build as a gorilla. A fossil mandible from a Gigantopithecus is found that is 9.5 in. Assuming that Gigantopithecus did have the same proportions as a gorilla estimate the body length and weight of the Gigantopithecus.

Use the munible to find the scaling factor

Estimated body length 141.4 in

Estimated weight 3513.3 lbs.

$\lambda = \frac{9.5}{4.5} = 2.11$

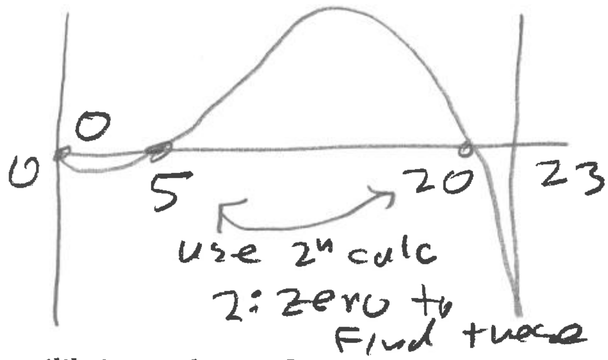
so scaled length is $67(\lambda) = 67(2.11) = 141.4$

scaled weight is $374(\lambda)^3 = 374(2.11)^3 = 3513.3$

5. (20 points) For the differential equation

$$\frac{dP}{dt} = -0.37P^3 + 9.25P^2 - 37P$$

(a) Make a graph of $\frac{dP}{dt}$ as a function of P (that is P is on the horizontal axis and P' is on the vertical axis). *Hint:* Use Xmin=0 and Xmax=23.



(b) What are the equilibrium solutions?

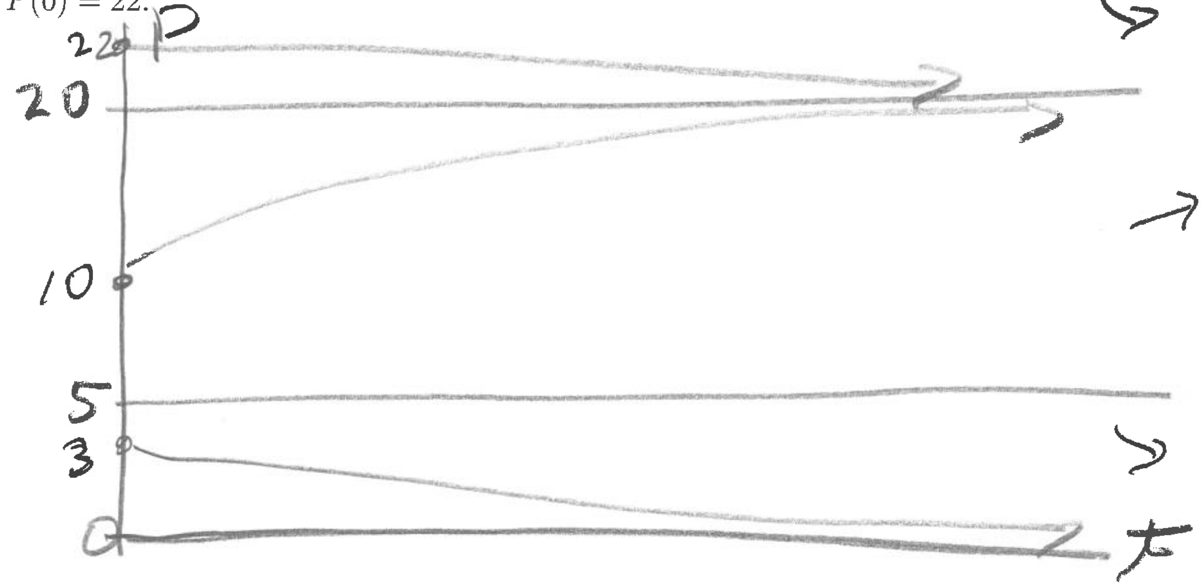
The equilibrium solutions are 0, 5, 20

(c) Which of the equilibrium solutions are stable?

The stable points are 0, 20

The stable points are where slope is negative (i.e. down hill)

(d) Make a graph of the time series of the solutions (t on the horizontal axis and P on the vertical axis) showing the equilibrium solutions along with the solutions with $P(0) = 3$, $P(0) = 10$ and $P(0) = 22$.



(e) It $P(0) = 10$ estimate $P(123)$. $P(123) \approx$ 20

starting at 10 the solution has the asymptote P=20 so for any large t P(t) ≈ 20.

6. (15 points) A backward pond is set up and before any fish are added a population of duckweed is started which grows logistically with an intrinsic growth rate of $r = 0.8$ lbs/day and a carrying capacity of $K = 3.4$ lbs. Let $P(t)$ be the number of pounds of duckweed in the pond after t days.

(a) What is the rate equation satisfied by $P(t)$?

The logistic equation is $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$

Rate equation is $\frac{dP}{dt} = .8P\left(1 - \frac{P}{3.4}\right)$

(b) After the duckweed population as settled down to its carrying capacity some gold fish are added to the pond and they eat the duckweed at a constant rate of 0.4lbs/day. What is the new rate equation satisfied by $P(t)$?

New rate equations is $\frac{dP}{dt} = .8P\left(1 - \frac{P}{3.4}\right) - .4$

(c) What is the new sable size of the duckweed population?

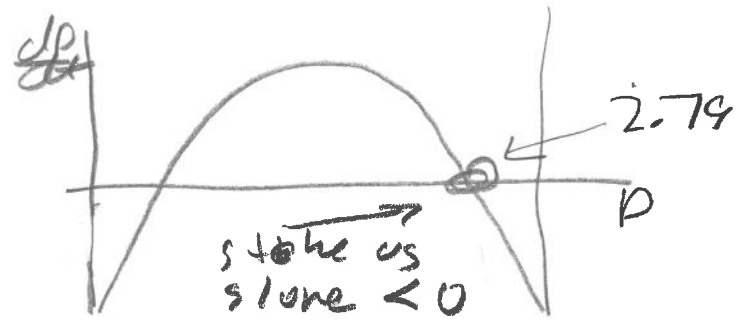
Stable size is 2.79

Plot $\frac{dP}{dt}$ as function of P

$Y1 = .8X(1 - X/3.4) - .4$

$X_{min} = 0$

$X_{max} = 3.4$



7. (10 points) A population of fish in a lake is overfished so that the intrinsic growth rate of the population is the constant $r = -0.4$ fish/year.

(a) If is originally stocked with 20,000 fish the give a formula for the number, $N(t)$, of fish after t years.

$N(t) = N_0 e^{rt}$

$= 20000 e^{-.4t}$

$N(t) =$ $20000 e^{-.4t}$

(b) DNR (Department of Natural Resources) starts stocking the lake at a constant rate of 1,000 fish/year. Then the new rate equation for N is

$$\frac{dN}{dt} = -0.4N + 1,000.$$

What is the stable size of the resulting fish population.

Stable population size is 2,500 fish.

The stationary solution is when

$\frac{dN}{dt} = -.4t + 1000 = 0$

$.4t = 1000$

$t = \frac{1000}{.4} = 2500$