Mathematics 172 Test 2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) A lake in a state park is overfished to the point that the intrinsic growth rate is $r = -.2$ (fish/year)/fish. The Department of Natural Resources wants a stable population of 5,000 fish in the lake. At what yearly rate should the pond be stocked?

Stocking rate is 1000 fish /year

Let $P(t)$ = size of fish population in year t.
S = stocking rate

Then
$$\frac{dP}{dt} = -.2P + S$$
We want $P = 5000$ to be a equilibrium point

so
$$0 = \frac{dP}{dt} = -.2(5000) + S$$
$$S = .2(5000) = 1000$$

2. (15 points) A population of algae grows in a fish tank. Let $A(t)$ be the number of grams of algae in the tank after t days. Assume that this grows logistically with an intrinsic growth rate of $r = .3$ (grams/day)/gram and a carrying capacity of $K = 150$ grams.

(a) Write the rate equation satisfied by $A(t)$.
The equation is $\frac{dA}{dt} = .3A\left(1 - \frac{A}{150}\right)$

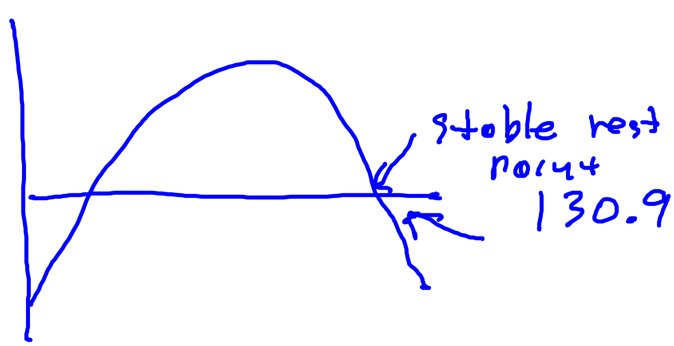
(b) A plecostomus, a type of algae eating catfish, is added to the tank which eats the algae at a constant rate of 5 grams/day. What is the new rate equation satisfied by $A(s)$.
The new equation is $A' = .3A\left(1 - \frac{A}{150}\right) - .5$

(c) What is the new stable population size of the algae population after the introduction of the ~~snail~~? plecostomus

Stable population size is 130.9 grams algae

$\backslash Y1 = .3X(1 - X/150) - 5$
$X_{min} = 0$
$X_{max} = 150$

2^{n} calc 2: zero



3. (10 points) Chickweed is a common weed in South Carolina. It is an annual and so has seeds once a year and then dies off. Assume in some park that one year there are 14 chickweeds and two years later there are 24. Assume that for the first 15 years that the growth rate of the chickweed is unconstrained.

(a) Find a formula for N_t, the number of chickweeds in the park after t years.

$N_t = N_0 \lambda^t = 14\lambda^t$

$N_t =$ $14(1.309)^t$ weeds

$N_2 = 14\lambda^2 = 24$ $\quad \lambda = \left(\frac{24}{14}\right)^{\frac{1}{2}} = 1.309$

$\lambda^2 = \frac{24}{14}$

(b) What is the per capita growth rate of the chickweeds?

$r =$ $\lambda - 1 = .309$

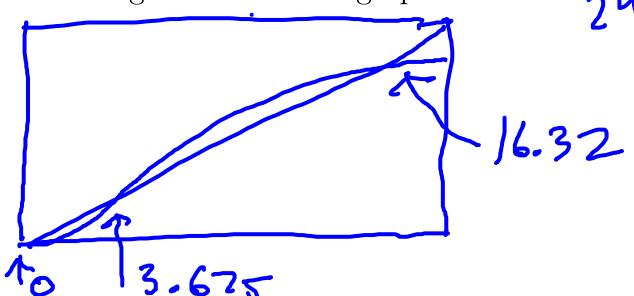
4. (15 points) Let us use a different model for chickweed growth. Let N_t be the number of chickweeds in the park in year t and assume

$$N_{t+1} = \frac{.4N_t + .2N_t^2}{1 + 0.01N_t^2}$$

Plot `\Y1 = (.4X+.2X^2)/(1 + .01X^2)` and `\Y2=X` with `Xmin=0` and `Xmax=20`.

(a) Make a rough sketch of the graphs here.

2nd calc 5: intersect



(b) What are the equilibrium points?

The equilibrium are 0, 3.675, 16.32

(c) Which of the equilibrium points are stable?

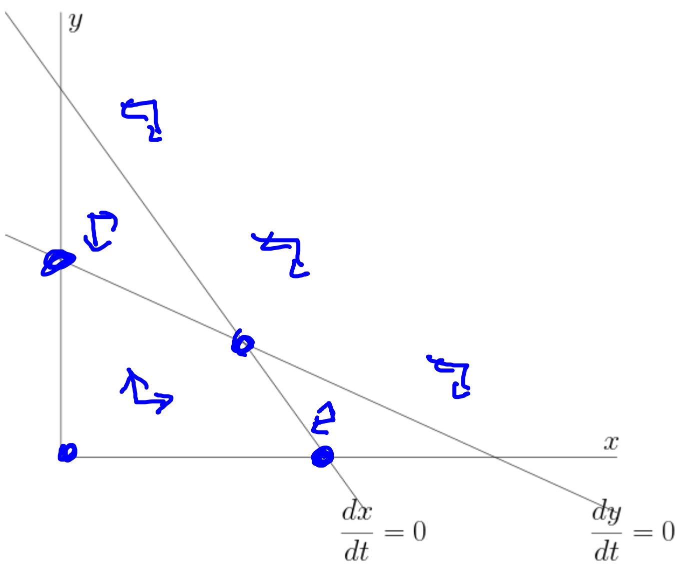
The stable points are 0, 16.32

Stable when |slope| < 1

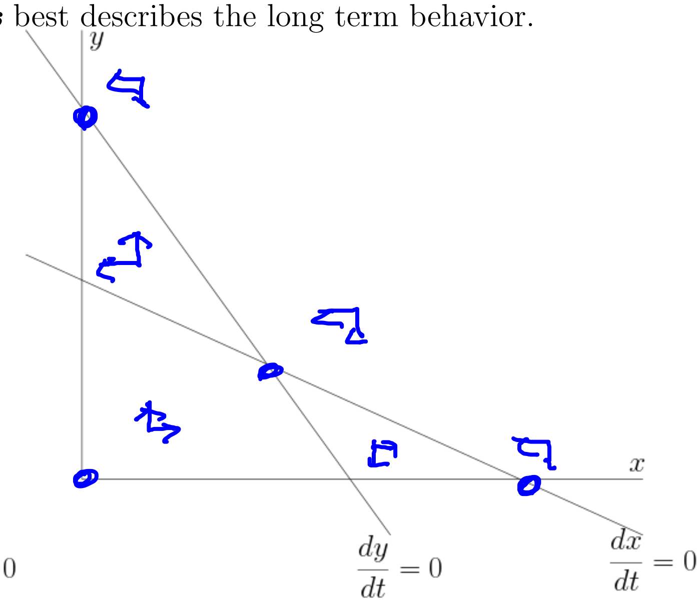
5. (20 points) Rabbits and ground squirrels live in a valley and compete for the same resources. Let $x(t)$ be the number of rabbits and $y(t)$ the number of squirrels in the valley after t years. We assume these satisfy our basic model for competing species:

$$\frac{dx}{dt} = r_1 x\left(\frac{K_1 - x - \alpha y}{K_1}\right)$$
$$\frac{dy}{dt} = r_2 y\left(\frac{K_2 - \beta x - y}{K_2}\right)$$

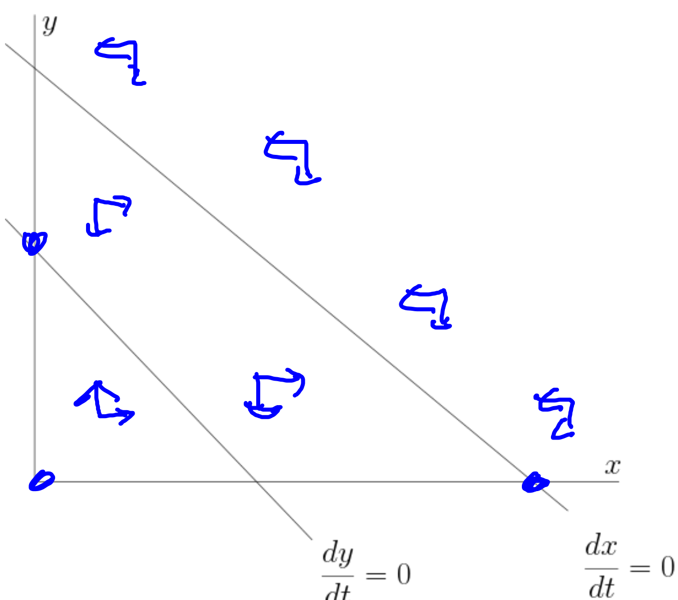
For each of the following label the equilibrium points with a large dot, ●, draw in the arrows showing which way x and y are moving, label which of the equilibrium points are stable, and finally label which of the four cases we are in, ***completive coexistence***, ***completive exclusion***, x ***species dominates***, or y ***species dominates*** best describes the long term behavior.



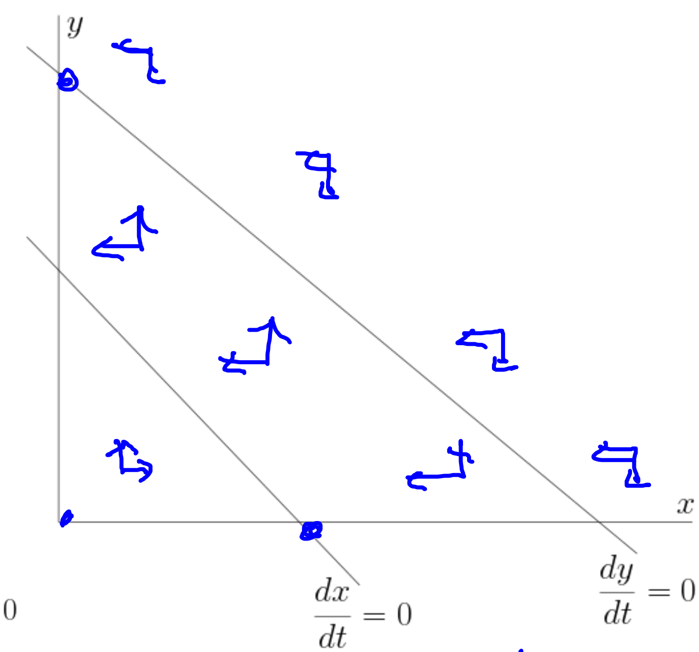
Long term behavior Coexistance



Long term behavior exclusion



Long term behavior x-dominates

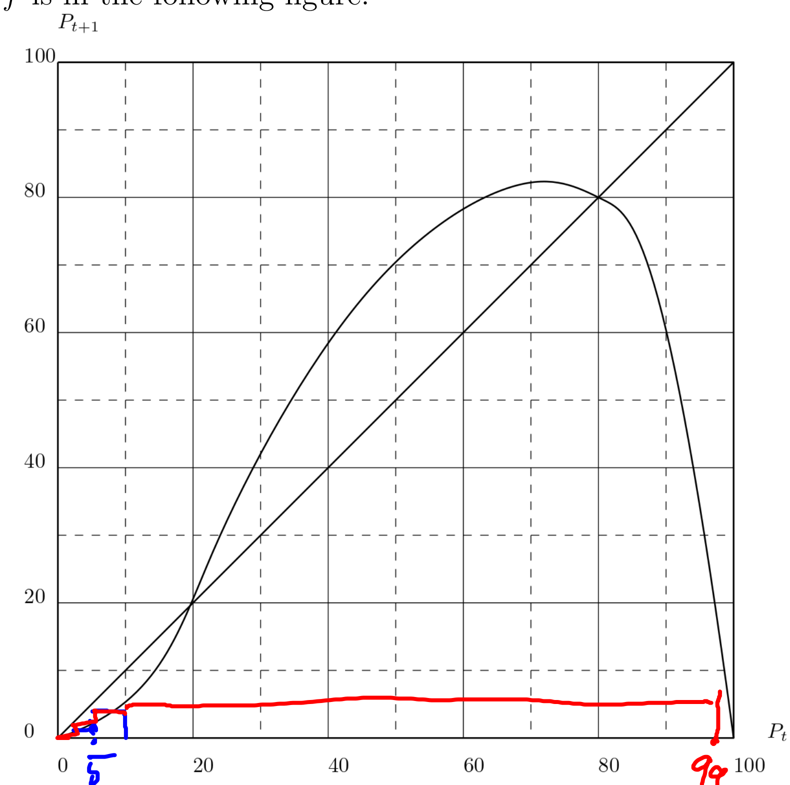


Long term behavior y-dominates

6. (20 points) Let P_t be a population that grows by the rule

$$P_{t+1} = f(P_t)$$

where the graph of f is in the following figure.



(a) What are the equilibrium points?

The equilibrium points are 0, 20 80

(b) What are the stable equilibrium points?

The stable points are: 0, 80

(c) How did you determine how the points are stable? (Write a sentence or two explaining.)

|slope| < 1

(d) If $P_0 = 10$. Estimate the following:

$P_1 \approx$ 5 $\quad$ $P_2 \approx$ 2 $\quad$ $p_{100} \approx$ 0

(e) If $P_0 = 80$. Estimate the following:

$P_1 \approx$ 80 $\quad$ $P_2 \approx$ 80 $\quad$ $p_{100} \approx$ 80

(f) If $P_0 = 99$ estimate P_{123}. $\quad$ $P_{123} \approx$ 0

7. (10 points) For the system of rate equations

$$\frac{dx}{dt} = .1x(1 - .1x - .2y)$$
$$\frac{dy}{dt} = .2y(1 - .5x - .3y)$$

if $x(10) = 1$ and $y(10) = 2$ estimate $x(10.2)$ and $y(10.2)$

$x(10.2) \approx$ 1.01 $\qquad$ $y(10.2) \approx$ 1.992

$x'(10) = .1(1)(1 - .1(1) - .2(2)) = .05$

$y'(10) = .2(2)(1 - .5(1) - .3(2)) = -.04$

$x(10.2) \approx x(10) + x'(10)(.2) = 1 + (.05)(.2) = 1.01$

$y(10.2) \approx y(0) + y'(10)(.2) = 2 + (-.04)(.2) = 1.992$