Mathematics 554 Homework.

In *Notes on analysis* do problems do Problems 2.36–2.45. These will be due on Monday.

The following is a good exercise in working with the definition of the least upper bound.

Problem 1. Let A and B be subsets of $\mathbb{R}$ that are each bounded above. Let

$$S = A + B = \{a + b : a \in A \text{ and } b \in B\}$$

(a) Show that S is bounded above.
(b) Prove

$$\sup(S) = \sup(A) + \sup(B).$$

□

We are working ourselves up to the first hard theorem of the course: Theorem 2.53 (Lipschitz intermediate value theorem) and some of the problems involve the definition and basic properties of Lipschitz function.

Depending how much we cover on Wednesday and Friday I may add Problem 2.46 (the proof of the Lipschitz intermediate value theorem). This one problem will involve as much work as all the other problems above. □

Problem 2. Let $S \subseteq \mathbb{R}$ be a subset that satisfies the two conditions
(a) S is bounded above.
(b) If $s_1, s_2 \in S$ with $s_1 \neq s_2$, then

$$|s_1 - s_2| \geq 1.$$

Show $\sup(S) \in S$ and therefore S has a maximum.