Mathematics 300 Homework, October 30, 2024.

Assume that McVeggies sells McBrocolli nuggets in containers of size 3 or 5 nuggets. Let us see for what values of n it is possible to buy n nuggets.

n	Possible?	How?
1	No	
2	No	
3	Yes	1(3 nuggets)
4	No	
5	Yes	1(5 nuggets)
6	Yes	2(3 nuggets)
8	Yes	1(3 nuggets) + 1(5 nuggets)
9	Yes	3(3 nuggets)
10	Yes	2(5 nuggets)
11	Yes	2(3 nuggets) + 1(5 nuggets)
12	Yes	4(3 nuggets)
13	Yes	1(3 nuggets) + 2(5 nuggets)

This makes is look like we can do n with $n \geq 5$. The idea to do this is to assume (this is our induction hypothesis) we that we can buy exactly n nuggets and use this to show we get $(n+1$ nuggets.

More precisely we either take on a size 5 package and add in two size 3 packages or take out three three size 3 packages and add in two size 5. Either case increases the total by 1 for a total of $(n+1)$ nuggets. Note that since we may have to take out as much as three size three packages $n \geq 9$ to make the induction step work.

Proposition 1. *Using only size* 3 *and size packages we can buy exactly* n *McBrocolli nuggets for any* $n \geq 5$

Proof. **Base cases:** From the table we see it is possible buy exactly n nuggets for $n = 5, 6, 7, 8, 9$. So we only have to consider the cases with $n \geq 9$.

Induction step: Assume we can realize exactly n nuggets and that $n \geq 9$. Our **induction goal** is to show that we can realize exactly $(n+1)$ nuggets.

Case 1. In getting n nuggets, there was at least one size 5 package. Then return the size 5 package and replace it with two size 3 packages. We now have $n - 5 + 2(3) = n + 1$ nuggets and we have reached our goal.

Case 2. There are no size 5 package used to get the packages of n nuggets. Then all that packages are of size 3. Then, as $n \geq 9$, there are at least three size 3 packages. Take out three size 3 packages and replace them two size 5 packages for a new total of $n - 3(3) + 2(5) = n + 1$ nuggets. So we have reached our goal in this case also. □

Problem 1. If the packages only come in sizes of 2 nuggets and 3 nuggets, for what n is is possible to buy exactly n nuggets? Make a table and a conjecture and then prove your conjecture using induction.

Problem 2. Use induction to prove

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

for all $n \geq 1$.