Mathematics 300 Homework, November 13, 2024.

Recall the ***empty set***, denoted $\varnothing$ s the set with no elments. Therefore

$$\varnothing = \{\}.$$

Problem 1. Explain why $\varnothing \neq \{\varnothing\}$. □

As menioned in class $\varnothing$ is a subset of every set. That is $\varnothing \subseteq A$ for all sets A. This is because

$$x \in \varnothing \quad \Longrightarrow \quad x \in A$$

for all A as the statement $x \in \varnothing$ is always false and an implication with false premise is true.

Two sets, A and B, are ***disjoint*** if and only if $A \cap B = \varnothing$. That is if they have no elements in common.

Problem 2. In the text do problems 1–7 on pages 238–240.

Problem 3. Let $p(x)$ and $q(x)$ be polynomials and set $f(x) = p(x)q(x)$. Let

$$P = \{r \in \mathbb{R} : p(r) = 0\} \qquad \text{(the set of roots of } p(x)\text{)}$$
$$F = \{r \in \mathbb{R} : f(r) = 0\} \qquad \text{(the set of roots of } f(x)\text{)}$$

Prove $P \subseteq F$. □

Problem 4. Let

$$A = \{x \in \mathbb{Z} : x \equiv 3 \pmod 4\}$$
$$B = \{12x + 8y + 7 : x, y \in \mathbb{Z}\}.$$

Prove $A = B$.