# Mathematics 300 Homework, September 7, 2024.

**Problem** 1. For more practice in taking negations of sentences with quantifiers do problems 12 and 14 on pages 77–79 of the text.

We are now about to start do proofs about basic number theory. This requires some definitions you have to have memorized. On the quiz on Monday you will have to state the following definition:

**Definition** (Text, page 82)**.** A nonzero integer $m$ ***divides*** an integer n provided that there is an integer $q$ such that $n = qm$.

If $m$ divides $n$ then we also say that $m$ is a ***factor*** of $n$ and that $n$ is a ***multiple*** of $m$.

**Notation.** We write $m \mid n$ for "$m$ divides $n$".

**Problem** 2. Show that $-3 \mid -42$.

*Solution.* This is nothing more than knowing the definition and basic arithmetic.

$$-72 = (-24)(-3) = q(-3)$$

where $q = -24$ is an integer. □

**Problem** 3. Show $m \mid 0$ for all integers $m \neq 0$.

*Solution.* Just note

$$0 = 0 \cdot m = qm$$

where $q = 0$ is an integer. □

**Problem** 4. Show that that if $a \in \mathbb{Z}$ and $a \neq -1$, then $(a + 1) \mid (a^2 - 1)$.

*Solution.* This one is more fun as we get to do some factoring:

$$\begin{aligned} a^2 - 1 &= (a - 1)(a + 1) \\ &= q(a + 1) \end{aligned}$$

where $q = a - 1$ is an integer and thus we have verified that definition of $(a+1) \mid a^2-1$. The restriction of $a \neq -1$ is that if $a = -1$, then $(a+1) = 0$ and under our defintion zero is not a divisor of anything. □

**Problem** 5. Show the product of any two even integers is divisible by 4.

*Solution.* Let $a$ and $b$ be even integers. We are to show $4 \mid ab$. We recall that $a$ and $b$ being even means there are integers $s$ and $t$ (or what every variable names you wish to use) so that

$$a = 2s, \qquad b = 2t.$$

Then the product of $a$ and $b$ is

$$ab = (2s)(2t) = 4(st) = 4q$$

where $q = st$ is an integer. Thus $4 \mid ab$ as required. □