

Quiz 1

Name: Kex*You must show your work to get full credit.*1. Let x be an integer(a) Define x is even. if and only if $x = 2q$ for some $q \in \mathbb{Z}$ (b) Define x is odd. if and only if $x = 2q + 1$ for some $q \in \mathbb{Z}$ 2. Prove or give a counterexample: If x is even, then $x^2 + 1$ is odd.Proof Assume x is even, then $x = 2q$ for some $q \in \mathbb{Z}$.

$$\begin{aligned}
 \text{Then } x^2 + 1 &= (2q)^2 + 1 \\
 &= 4q^2 + 1 \\
 &= 2(2q^2) + 1 \\
 &= 2a + 1
 \end{aligned}$$

where $a = 2q^2 \in \mathbb{Z}$ by closure under multiplication.Thus $x^2 + 1$ is odd3. Prove or give a counterexample: If x is even, then $x^3 + 5$ is odd.Proof Assume x is even. Then $x = 2q$ for some $q \in \mathbb{Z}$.

$$\begin{aligned}
 \text{So } x^3 + 5 &= (2q)^3 + 5 \\
 &= 8q^3 + 5 \\
 &= 8q^3 + 4q + 1 \\
 &= 2(4q^3 + 2q) + 1 \\
 &= 2n + 1
 \end{aligned}$$

where $n = 4q^3 + 2q \in \mathbb{Z}$ by closure under addition.Thus $x^3 + 5$ is odd.