Mathematics 300

Quiz 1 **Name:** Key

You must show your work to get full credit.

1. Let

$$S = a + ar + ar^2 \cdots + ar^n$$

Derive a formula for this sum when $r \neq 1$.

$S = \dfrac{a - ar^{n+1}}{1 - r}$ (Mult. by r)

$S = a + ar + ar^2 + \cdots + ar^n$

$rS = ar + ar^2 + \cdots + ar^n + ar^{n+1}$

$S - rS = a - ar^{n+1}$

$(1-r)S = a - ar^{n+1}$

$S = \dfrac{a - ar^{n+1}}{1-r} = \dfrac{\text{first} - \text{next}}{1 - \text{ratio}}$

2. Use that $11 \equiv -1 \pmod{10}$ to show that the number with decimal $abcd$ is divisible by 11 if and only if $d - c + b - a$ is divisible by 11.

Let $n = abcd = a(10)^3 + b(10)^2 + c(10) + d$.

Then

$n \equiv a(10)^3 + b(10)^2 + c(10) + d \pmod{11}$

$\equiv a(-1)^3 + b(-1)^2 + c(-1) + d \pmod{11}$

$\equiv -a + b - c + d \pmod{11}$

So $n \equiv 0 \pmod{11} \iff d - c + b - a \equiv 0 \pmod{11}$

I.e. $11 \mid n \iff 11 \mid (d - c + b - a)$.

3. Find the sum of

$$S = 1 + 3 + 3^2 + 3^3 + \cdots + 3^{20}.$$

The sum is $\dfrac{3^{21} - 1}{2}$

$S = \dfrac{\text{first} - \text{next}}{1 - \text{ratio}}$

$= \dfrac{1 - 3^{21}}{1 - 3} = \dfrac{1 - 3^{21}}{1 - 3} = \dfrac{3^{21} - 1}{2}$