Mathematics 300

**Quiz 14** **Name:** Key

***You must show your work to get full credit.***

**1.** Prove: if $r$ is a rational number, then so is $\dfrac{r^2-2}{r^2+1}$.

Assume $r$ is rational. Then $r = \frac{p}{q}$ with $p, q$ integers and $q \neq 0$.

Then
$$\frac{r^2-2}{r^2+1} = \frac{(\frac{p}{q})^2-2}{(\frac{p}{q})^2+1}$$
$$= \frac{((\frac{p}{q})^2-2)q^2}{((\frac{p}{q})^2+1)q^2}$$
$$= \frac{p^2-2q^2}{p^2+q^2} = \frac{A}{B}$$

where $A = p^2-2q^2$, $B = p^2+q^2$ are integers by closure properties of the integers.

**2.** What is the sum $S = 1 + 3 + 3^2 + \cdots + 3^n$. $S = \dfrac{3^{n+1}-1}{2}$

This is a geometric series so

$$S = \frac{\text{first} - \text{next}}{1 - \text{ratio}}$$
$$= \frac{1-3^{n+1}}{1-3}$$
$$= \frac{1-3^{n+1}}{-2}$$
$$= \frac{3^{n+1}-1}{2}$$