Mathematics 300

Quiz 15 Name: Key

You must show your work to get full credit.

Proposition. *If n is an integer and n^2 is even, then n is even.*

1. Use this proposition to prove $\sqrt{2}$ is irrational.

Towards a contradiction assume that $\sqrt{2}$ is rational. Then

(1) $\sqrt{2} = \frac{p}{q}$

where $p, q \in \mathbb{Z}$. We can, and do, assume this is in lowest terms, that is p and q have no common factor.

Multiply (1) by q to get

$q\sqrt{2} = p$.

Square this to get

(2) $2q^2 = p^2$

As q^2 is an integer, this implies p^2 is even. By the proposition this implies p is even, so $p = 2a$ for some integer a. Put this in equation (2)

$2q^2 = (2a)^2$

so $2q^2 = 4a^2$

$q^2 = 2a^2$.

Thus q^2 is even and we can again use the proposition to get that q is even. So $q = 2b$ for some $b \in \mathbb{Z}$.

Therefore

$\frac{p}{q} = \frac{2a}{2b}$

which contradicts that $\frac{p}{q}$ is in lowest terms.