Mathematics 300

Quiz 16 **Name:** Key

You must show your work to get full credit.

In doing these problems you are allowed to use that the set of rational numbers is closed under addition, subtraction, multiplication, and (when defined) division.

1. If r and s are rational numbers with $r \neq 0$, show $\dfrac{s-r-1}{r}$ is rational.

By closure of the rational numbers under subtraction, $s-r \in \mathbb{Q}$. As 1 is rational we can again use closure under substraction to get $(s-r)-1 = s-r-1 \in \mathbb{Q}$. Final we can use closure under division (as $r \neq 0$) to conclude

$$\frac{s-r-1}{r} \in \mathbb{Q}$$

as required

2. Let θ be irrational and r rational with $r \neq 0$. Show $s = r\theta + r + 1$ is irrational.

Towards a contradiction assue s is rational. We solve for θ in $s = r\theta + r + 1$

$$s - r - 1 = r\theta$$
$$\theta = \frac{s-r-1}{r}.$$

By problem 1, $\frac{s-r-1}{r}$ is rational, contradicting that θ is irrational.