Mathematics 300

Quiz 17 Name: Key

You must show your work to get full credit.

1. Prove for any integer n that the product $(n+2)(n-3)$ is even.

Proof 1 There are two cases: n even or n odd.

case 1 n even. Then $n = 2q$ for some $q \in \mathbb{Z}$.

So $(n+2)(n-3) = (2q+2)(2q-3)$

$= 2(q+1)(2q-3)$

$= 2k$

where $k \in \mathbb{Z}$, so $(n+2)(n-3)$ is even in this case.

case 2 n is odd. Then $n = 2q+1$ for some $q \in \mathbb{Z}$.

Then

$(n+2)(n-3) = (2q+1+2)(2q+1-3) = (2q+3)(2q-2)$

$= 2k$

where $k = (2q+3)(q-1) \in \mathbb{Z}$. so $(n+2)(n-3)$ is even in this case 2

Proof 2 We wish to show $(n+2)(n-3) \equiv 0 \pmod 2$

(This is equivalent to $(n+2)(n-3)$ being even.)

Case 1 $n \equiv 0 \pmod 2$

Then $(n+2)(n-3) \equiv (0+2)(0-3) = -6 \equiv 0 \pmod 2$.

case 2 $n \equiv 1 \pmod 2$

Then

$(n+2)(n-3) \equiv (1+2)(1-3) = -6 \equiv 0 \pmod 2$.

So in all cases $(n+2)(n-3) \equiv 0 \pmod 2$.

Thus $(n+2)(n-3)$ is even.