Mathematics 300

Quiz 18 **Name:** Key

You must show your work to get full credit.

1. Show that for all integers n that $n^3 - n^2$ is even.

It is easiest to work (mod 2). So we will show $n^3 - n^2 \equiv 0 \pmod 2$ for all $n \in \mathbb{Z}$.

There are two cases:

Case 1 $n \equiv 0 \pmod 2$.

Then $n^3 - n^2 \equiv 0^3 - 0^2 \pmod 2$
$\equiv 0 \pmod 2$

Case 2 $n \equiv 1 \pmod 2$

Then
$n^3 - n^2 \equiv 1^3 - 1^2 \pmod 2$
$\equiv 0 \pmod 2$

So $n^3 - n^2 \equiv 0$ in all cases and thus $n^3 - n^2$ is even for all n.

2. Define for any real number x define:

$$\operatorname{sgn}(x) = \begin{cases} 1, & x > 0; \\ 0, & x = 0; \\ -1, & x < 0. \end{cases}$$

Show for $x \neq 0$ that $\frac{x}{|x|} = \operatorname{sgn}(x)$.

Let $x \neq 0$. Then $x > 0$ or $x < 0$. So we have two cases.

Case 1 $x > 0$. Then $|x| = x$, thus

$$\frac{x}{|x|} = \frac{x}{x} = 1 = \operatorname{sgn}(x) \quad \text{(by def. of sgn(x))}$$

Case 2 $x < 0$. Then $|x| = -x$. So this time

$$\frac{x}{|x|} = \frac{x}{-x} = -1 = \operatorname{sgn}(x) \quad \text{(by def. of sgn(x))}$$

So we have $\operatorname{sgn}(x) = \frac{x}{|x|}$ for all $x \neq 0$.