Mathematics 300

Quiz 2 Name: Key.

You must show your work to get full credit.

1. Prove or give a counterexample: If x is odd, then $3x^2 - 1$ is also odd.

counterexample: $x = 1$ is odd, but
$3x^2 - 1 = 3(1)^2 - 1 = 3 - 1 = 2$ is not odd.

2. An integer x is of ***type 0*** if and only if $x = 3n$ for some integer n, x is of ***type 1*** if and only if $x = 3n + 1$ for some integers n, x is of ***type 2*** if and only if $x = 3n + 2$ for some integer n. Prove or give a counterexample: If x is of type 2, then $x^2 - 2$ is also of type 2.

Let us try a few type numbers
$x = 2$, then $x^2 - 2 = 2^2 - 2 = 2$. is type 2
$x = 5$, then $x^2 - 2 = 25 - 2 = 23 = 3(7) + 2$ is of type 2

So it seems to be true.

Proof Assume x is of type 2. Then $x = 3q + 2$ for some $q \in \mathbb{Z}$.

$$\begin{aligned} x^2 - 2 &= (3q+2)^2 - 2 \\ &= 9q^2 + 6q + 4 - 2 \\ &= 9q^2 + 6q + 2 \\ &= 3(3q^2 + 2q) + 2 \\ &= 3n + 2 \end{aligned}$$

where $n = 3q^2 + 2q \in \mathbb{Z}$ by closure properties of $\mathbb{Z}$. Thus $x^2 - 2$ is of type 2.