Mathematics 300

Quiz 20 **Name:** Key

You must show your work to get full credit.

1. Show that for any integer n that $3 \mid n(n-1)(n-2)$.

Here it is easiest to work (mod 3).

Case 1 $n \equiv 0 \pmod 3$.
Then $n(n-1)(n-2) \equiv 0(0-1)(0-2) \equiv 0 \pmod 3$

Case 2 $n \equiv 1 \pmod 3$
Then $n(n-1)(n-2) \equiv 1(\underbrace{1-1}_{=0})(1-2) \equiv 0 \pmod 3$

Case 3 $n \equiv 2 \pmod 3$
Then $n(n-1)(n-2) \equiv 2(2-1)(\underbrace{2-2}_{=0}) \equiv 0 \pmod 3$

So $n(n-1)(n-2) \equiv 0$ for all n. This implies
$3 \mid n(n-1)(n-2)$ for all n.

2. Show that if a, b, and c are odd integers then, for any integer n the number $an^2 + bn + c$ is odd.

As a, b, c are odd $a, b, c \equiv 1 \pmod 2$. Now there are two cases.

Case 1 $n \equiv 0 \pmod 2$
Then
$an^2 + bn + c \equiv 1(0)^2 + 1(0) + 1 \equiv 1 \pmod 2$
so $an^2 + bn + c$ is odd in this case.

Case 2 $n \equiv 1 \pmod 2$
Then
$an^2 + bn + c \equiv 1(1)^2 + 1(1) + 1 \equiv 3 \equiv 1 \pmod 2$
so $an^2 + bn + c$ is also odd in this case.