Quiz 21

Name: Key

You must show your work to get full credit.

- 1. State or define the following:
 - (a) For $a \mid b$ for integers a and b. \iff $a \neq 0$, and b = qa for some 14 teger 91.
 - (b) For $a \equiv b \pmod{n}$ for integers a, b, n. N31 and those is an integer of so that a-b=96
 - Let a_ib be in legers with $b \geqslant l$. Then there are unique in legers q, r > 0 that a = qb + r and $0 \le r < b$. (c) The division algorithm.
 - (d) The real number a is rational. $\Leftrightarrow a = \frac{1}{4}$ with $mq \in \mathbb{Z}$ and of \$ 0.
 - (e) The real number a is irrational. (e) a 15 not a various various
- 2. Find the sum of the geometric series

$$S = \frac{1 - 2 + 4 - 8 + 18 - \dots + 2^{10}}{1 - \frac{1 - 2^{11}}{1 - \frac{1 - 2^{11}}{1 - 1}}} = \frac{2^{11} - 1}{1 - \frac{1 - 2^{11}}{1 - 1}} = \frac{2^{11} - 1}{1 - \frac{1 - 2^{11}}{1 - \frac{1 - 2^{11}}{1 - 1}}} = \frac{2^{11} - 1}{1 - \frac{1 - 2^{11}}{1 - 1}} = \frac{2^{11} - 1}{1 - \frac{1 - 2^{11}}{1 - 1}} = \frac{2^{11$$

3.	What is the remainder when 9,437 is divided by 3?
	The remainder is

4. Prove or give a counterexample: If a and n are positive integers and $a \mid n^2$, then $a \mid n$.

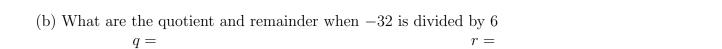
5. Prove for any integer n that $3 \mid n^2$ implies $3 \mid n$.

6. Use pro	oblem 5 to s	how $\sqrt{3}$ is	irrational.	
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7. Prove directly from the definition of rational that if r is a rational number, then so is	$\frac{r}{1+r^2}.$
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8. Prove that if α is irrational and r is rational and $r \neq 0$, then $r\alpha + r - 3$ is irrational. (For this problem you can use that the basic closure properties of \mathbb{Q} , that is that \mathbb{Q} is closed under addition, subtraction, multiplication, and division (when defined).)

9.	9. Prove that for any integer n that $n(n+4)(n+5)$ is divisible by 3.								
10	• (a) What are the quotient and remainder when 32 is divided by 6								



11. Prove or give a counterexample: if a and b are both irrational, then a + b is irrational.

12. Prove or give a counterexample: If a^2 is irrational, then a is irrational.