Mathematics 300

Quiz 21 Name: Key

You must show your work to get full credit.

1. State or define the following:

(a) For $a \mid b$ for integers a and b. $\iff$ $a \neq 0$, and $b = qa$ for some integer q.

(b) For $a \equiv b \pmod{n}$ for integers a, b, n. $\iff$ $n \geq 1$ and there is an integer q so that $a - b = qn$.

(c) The ***division algorithm***. Let a, b be integers with $b \geq 1$. Then there are unique integers q, r so that $a = qb + r$ and $0 \leq r < b$.

(d) The real number a is ***rational***. $\iff$ $a = \frac{p}{q}$ with $p, q \in \mathbb{Z}$ and $q \neq 0$.

(e) The real number a is ***irrational***. $\iff$ a is not a rational number

2. Find the sum of the geometric series

$$S = 1 - 2 + 4 - 8 + 18 - \cdots + 2^{10}$$

$$S = \frac{\text{first} - \text{next}}{1 - \text{ratio}} = \frac{1 - 2^{11}}{1 - 2} = \frac{1 - 2^{11}}{-1} = 2^{11} - 1$$

$S =$ $2^{11} - 1 = 2048$

3. What is the remainder when 9,437 is divided by 3?

The remainder is ________________________

4. Prove or give a counterexample: If a and n are positive integers and $a \mid n^2$, then $a \mid n$.

5. Prove for any integer n that $3 \mid n^2$ implies $3 \mid n$.

6. Use problem 5 to show $\sqrt{3}$ is irrational.

7. Prove directly from the definition of rational that if r is a rational number, then so is $\dfrac{r}{1+r^2}$.

8. Prove that if α is irrational and r is rational and $r \neq 0$, then $r\alpha + r - 3$ is irrational. (For this problem you can use that the basic closure properties of $\mathbb{Q}$, that is that $\mathbb{Q}$ is closed under addition, subtraction, multiplication, and division (when defined).)

9. Prove that for any integer n that $n(n+4)(n+5)$ is divisible by 3.

10. (a) What are the quotient and remainder when 32 is divided by 6

$q =$ ____________ $r =$ ____________

(b) What are the quotient and remainder when -32 is divided by 6

$q =$ ____________ $r =$ ____________

11. Prove or give a counterexample: if a and b are both irrational, then $a+b$ is irrational.

12. Prove or give a counterexample: If a^2 is irrational, then a is irrational.