Mathematics 300

Quiz 22 **Name:** Key

You must show your work to get full credit.

Let $y = x^2 e^x$. Use induction to show the n-th derivative of y is

$$\frac{d^n y}{dx^n} = \big(x^2 + 2nx + n(n-1)\big)e^x$$

In your proof be sure to state the base case that the induction step.

Base case: $y' = 2xe^x + x^2e^x$ (product rule)

$= (x^2+2x)e^x$

$= (x^2+2(1)x + 1(1-1))e^x$.

which show base case of $n=1$ holds.

Induction hypothesis $y^{(n)} = (x^2+2nx+n(n-1))e^x$.

Take the derivative using the product rule

$y^{(n+1)} = (2x+2n)e^x + (x^2+2nx+n(n-1))e^x$

$= (2x+2n + x^2+2nx+n(n-1))e^x$

$= (x^2 + (2n+2)x + (2n + n(n-1)))e^x$

$= (x^2 + 2(n+1)x + n(2+n-1))e^x$

$= (x^2 + 2(n+1)x + n(n+1))e^x$

$= (x^2 + 2(n+1)x + (n+1)((n+1)-1))e^x$

which closes the induction.