Mathematics 300

Quiz 23 Name: Key

You must show your work to get full credit.

We have seen that if $r \neq 1$ then

$$a + ar + ar^2 + \cdots + ar^n = \frac{a - ar^{n+1}}{1-r}.$$

Give a proof of this using induction.

Base case: $n=0$ $\quad a = \dfrac{a - ar^{0+1}}{1-r} = \dfrac{a(1-r)}{1-r} = a.$

So this holds.

Induction hypothesis:

$$a + ar + \quad + ar^n = \frac{a - ar^{n+1}}{1-r}$$

add ar^{n+1} to both sides

$$a + ar + \cdots + ar^n + ar^{n+1} = \frac{a - ar^{n+1}}{1-r} + ar^{n+1}$$

$$= \frac{a - ar^{n+1}}{1-r} + \frac{ar^{n+1}(1-r)}{1-r}$$

$$= \frac{a - \cancel{ar^{n+1}} + \cancel{ar^{n+1}} - ar^{n+2}}{1-r}$$

$$= \frac{a - ar^{n+2}}{1-r}$$

$$= \frac{a - ar^{(n+1)+1}}{1-r}$$

which closes the induction.