Mathematics 300

Quiz 27 Name: Key.

You must show your work to get full credit.

Recall that a positive integer p is ***prime*** if and only if $p \geq 2$ and the only positive divisors of p are 1 and p.

Theorem. *Every integer n is a product of primes*, $n \geq 2$.

1. Prove this.

Base case: $n=2$. 2 is a prime, so this case holds.

<u>Induction step.</u> Assume that for every k with $2 \leq k \leq n$, that k is a product of primes. We now show $(n+1)$ is a product of primes.

<u>Case 1</u> $n+1$ is prime. Then we are done in this case.

<u>Case 2</u> $n+1$ is not prime. Then $n+1$ factors $n+1 = ab$ where $a, b \in \mathbb{N}$ with $2 \leq a \leq n$, $2 \leq b \leq n$. Then by the induction hypothesis each of a and b are products of primes. Say

$$a = p_1 p_2 \cdots p_k$$
$$b = q_1 q_2 \cdots q_l.$$

Then $n+1 = ab = p_1 p_2 \cdots p_k q_1 q_2 \cdots q_l$ is a product of primes. <u>done.</u>

Theorem. *There are infinitely many primes.*

2. Prove this.