Mathematics 300

Quiz 34 Name: Key

You must show your work to get full credit.

Let $f\colon A \to B$ be a function between sets and $g\colon B \to A$ a function in the "other direction". Then g is the ***inverse*** of f if and only if

$$g(f(a)) = a \quad \text{for all } a \in A \qquad \text{and} \qquad f(g(b)) = b \quad \text{for all } b \in B.$$

1. Prove: if $f\colon A \to B$ has an inverse, then f is surjective.

Let $b \in B$. Then $f(g(b)) = b$. Thus
$f(x) = b$ has the solution $x = g(b)$.
So $b \in$ range(f). This holds for
all $b \in B$. Thus f is surjective.

2. Prove: if $f\colon A \to B$ has an inverse, then f is injective.

Let $f(a_1) = f(a_2)$. We want to
show $a_1 = a_2$. Apply g to this
equation
$g(f(a_1)) = g(f(a_2))$.
Then $a_1 = a_2$. done

3. Prove: If ~~$f\colon AB$~~ $f\colon A \to B$, is bijective, then f has an inverse.

Let $b \in B$. As f is bijective (= one to one onto)
There is a unique $a \in A$ with
$f(a) = b$.
Define $g(b) = a$ where $f(a) = b$.
So $f(a) = b \iff g(b) = a$.
Thus $f(g(b)) = b$ and $g(f(a)) = a$ for
all $a \in A$, $b \in B$. So g is inverse of f.

4. Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = 5x - 20.$$

Find the inverse of f. The inverse is $g(y) = \frac{y+20}{5}$

Let $g(y)$ = inverse of $f(x)$

Then,

$y = f(g(y)) = 5g(y) - 20$

So $5g(y) - 20 = y$

$5g(y) = y + 20$

$g(y) = \frac{y+20}{5}$.

5. Prove that $n^2(n-1)(n+3)$ is divisible by 4 for all $n \in \mathbb{Z}$.

Let $N = n^2(n-1)(n+3)$.

To show N is even we consider 2 cases.

Case 1 n is even. Then $n = 2q$ for some $q \in \mathbb{Z}$. Then

$N = n^2(n-1)(n+3) = (2q)^2(n-1)(n+3)$

$= 4(q^2(n-1)(n+3))$

$= 4Q$

where $Q = q^2(n-1)(n+3) \in \mathbb{Z}$. so $4 \mid N$.

Case 2 n is odd. Then $n = 2q+1$ for some $q \in \mathbb{Z}$. Then

$N = n^2(2q+1-1)(2q+3-1) = n^2(2q)(2q+2)$

$= 4n^2q(q+1) = 4Q$

where $Q = n^2q(q+1) \in \mathbb{Z}$ so $4 \mid N$ in this case also.