Mathematics 300

Quiz 8 Name: Key

You must show your work to get full credit.

1. Prove or give a counter example: If a and b are integers with a even and b odd, then $8 \mid a^2(b+3)$.

This is true. Proof Assume a even and b odd. Then there are integers s, t such that

$$a = 2s \qquad b = 2t+1.$$

Thus

$$\begin{aligned} a^2(b+3) &= (2s)^2(2t+1+3) \\ &= 4s^2(2t+4) \\ &= 4s^2(2)(t+2) \\ &= 8s^2(t+2) \\ &= 8q \end{aligned}$$

where $q = s^2(t+2)$ is an integer. Thus $8 \mid a^2(b+3)$

2. Prove: If n^3 is odd, then so is n.

We prove the contrapositive: If n is even, then so is n^3. Assume n is even. Then $n = 2t$ for some integer t. Thus gives

$$n^3 = (2t)^3 = 8t^3 = 2(4t^3) = 2q$$

where $q = 4t^3$ is an integer. Thus n^3 is even.