Mathematics 300

Quiz 9 **Name:** Key

You must show your work to get full credit.

1. State the definition of ***m divides n***.

$m \mid n \iff m, n$ are integers, $m \neq 0$ and there is an integer q so that $n = qm$

2. State the definition of ***a is congruent to b modulo n***, that is of $a \equiv b \pmod{n}$.

$a \equiv b \pmod{m} \iff a, b, m \in \mathbb{Z}$, $m \geq 1$ and there exists a $q \in \mathbb{Z}$ with

$$a - b = qm.$$

3. Which of the following is true and why?

(a) $6 \equiv 8 \pmod{3}$ True or false? False

Why?

$6 - 8 = 2$ and 3 does not divide 2.

Thus $6 \not\equiv 8 \pmod{3}$

(b) $5 \equiv 13 \pmod{3}$ True or false? False

Why?

$5 - 13 = -8$ and 3 does not divide -8.

4. Prove that if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.

Assume $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$

Then there are $q_1, q_2 \in \mathbb{Z}$ such that

$$a - b = q_1 n$$
$$b - c = q_2 n.$$

Add these to get

$$a - b + b - c = q_1 n + q_2 n$$

$$a - c = (q_1 + q_2) n = q n$$

where $q = q_1 + q_2 \in \mathbb{Z}$. Thus

$$a \equiv c \pmod{n}$$

5. Prove that if $a \equiv b \pmod{n}$ and $x \equiv y \pmod{n}$, then $a + x \equiv b + y \pmod{n}$.

Assume $a \equiv b \pmod{n}$ and $x \equiv y \pmod{n}$.

Then there exist $q_1, q_2 \in \mathbb{Z}$ with

$$a - b = q_1 n$$
$$x - y = q_2 n.$$

Add these to get

$$(a+x) - (b+y) = q_1 n + q_2 n = (q_1 + q_2) n = q n$$

where $q = q_1 + q_2 \in \mathbb{Z}$. Thus

$$a + x \equiv b + y \pmod{n}.$$