Mathematics 300 Quiz 10 Name:______________________________

Show your work to get credit.

The problems are worth 10 points each.

1. Define the following:

(a) The integer n is ***even***.

(b) The integer n is ***odd***.

(c) The integer a ***divides*** the integer b (in symbols $a \mid b$).

(d) For integers a, b, n the a is ***congruent to*** b ***modulo*** n. (in symbols $a \equiv b \pmod{n}$.)

2. Prove or give a counterexample: If n is even, then $(n^3 + 11) \equiv 3 \pmod{8}$.

3. (a) Make truth tables for $P \to (P \to Q)$ and $\neg P \wedge Q$.

(b) Are $P \to (P \to Q)$ and $\neg P \wedge Q$ logically equivalent? Explain your answer.

4. Let P be the statement "If the number a is positive, then the equation $f(x) = a$ has a solution".
(a) What is the converse of this statement?

(b) What is the contrapositive of this statement?

(c) What is the negation of this statement?

5. What is the negation of the statement "Every even number is the sum of two prime numbers"?

6. (a) Write $\{x \in \mathbb{Z} : (2x+1)(x-4) = 0\}$ in roster notation. ______________________________

(b) $\{x \in \mathbb{Q} : (2x + 1)(x - 4) = 0\}$ in roster notation. ______________________________

(c) Write $\{-2, -1, 0, 1, 2, \ldots, 101, 102\}$ in set builder notation. ______________________________

7. Prove the transitive law for congruence. That is prove: If $r \equiv s \pmod{m}$ and $s \equiv t \pmod{m}$, then $r \equiv t \pmod{m}$.

8. Prove: If $a \equiv b \pmod{n}$ and $x \equiv y \pmod{n}$, then $a - x \equiv b - y \pmod{n}$.

9. Prove or give a counterexample: If $x \equiv y \pmod{n}$, then $x^2 \equiv y^2 \pmod{n}$.

10. Find all Pythagorean triples of the form m, $m + 3$, $m + 6$.

The triples are: ______________________________