

## Mathematics 300

### Quiz 21

Name: \_\_\_\_\_

*You must show your work to get full credit.*

1. State or define the following:

(a) For  $a \mid b$  for integers  $a$  and  $b$ .

(b) For  $a \equiv b \pmod{n}$  for integers  $a, b, n$ .

(c) The *division algorithm*.

(d) The real number  $a$  is *rational*.

(e) The real number  $a$  is *irrational*.

2. Find the sum of the geometric series

$$S = 1 - 2 + 4 - 8 + 18 - \cdots + 2^{10}$$

$S =$  \_\_\_\_\_

3. What is the remainder when 9,437 is divided by 3?

The remainder is \_\_\_\_\_

4. Prove or give a counterexample: If  $a$  and  $n$  are positive integers and  $a \mid n^2$ , then  $a \mid n$ .

5. Prove for any integer  $n$  that  $3 \mid n^2$  implies  $3 \mid n$ .

6. Use problem 5 to show  $\sqrt{3}$  is irrational.

7. Prove directly from the definition of rational that if  $r$  is a rational number, then so is  $\frac{r}{1+r^2}$ .

8. Prove that if  $\alpha$  is irrational and  $r$  is rational and  $r \neq 0$ , then  $r\alpha + r - 3$  is irrational. (For this problem you can use that the basic closure properties of  $\mathbb{Q}$ , that is that  $\mathbb{Q}$  is closed under addition, subtraction, multiplication, and division (when defined).)

**9.** Prove that for any integer  $n$  that  $n(n+4)(n+5)$  is divisible by 3.

**10.** (a) What are the quotient and remainder when 32 is divided by 6

$q =$  \_\_\_\_\_

$r =$  \_\_\_\_\_

(b) What are the quotient and remainder when  $-32$  is divided by 6

$q =$  \_\_\_\_\_

$r =$  \_\_\_\_\_

**11.** Prove or give a counterexample: if  $a$  and  $b$  are both irrational, then  $a + b$  is irrational.

**12.** Prove or give a counterexample: If  $a^2$  is irrational, then  $a$  is irrational.