You must show your work to get full credit.

- 1. State or define the following:
 - (a) For $a \mid b$ for integers a and b.

(b) For $a \equiv b \pmod{n}$ for integers a, b, n.

(c) The division algorithm.

(d) The real number a is rational.

(e) The real number a is irrational.

2. Find the sum of the geometric series

$$S = 1 - 2 + 4 - 8 + 18 - \dots + 2^{10}$$

3.	What is the remainder when 9,437 is divided by 3?
	The remainder is

4. Prove or give a counterexample: If a and n are positive integers and $a \mid n^2$, then $a \mid n$.

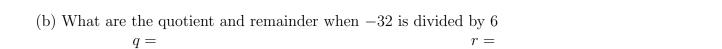
5. Prove for any integer n that $3 \mid n^2$ implies $3 \mid n$.

6. Use pro	oblem 5 to s	how $\sqrt{3}$ is	irrational.	
or one pre		now vons	iii wololiwi.	

7. Prove directly from the definition of rational that if r is a rational number, then so is	$\frac{r}{1+r^2}.$
--	--------------------

8. Prove that if α is irrational and r is rational and $r \neq 0$, then $r\alpha + r - 3$ is irrational. (For this problem you can use that the basic closure properties of \mathbb{Q} , that is that \mathbb{Q} is closed under addition, subtraction, multiplication, and division (when defined).)

9.	9. Prove that for any integer n that $n(n+4)(n+5)$ is divisible by 3.								
10	• (a) What are the quotient and remainder when 32 is divided by 6								



11. Prove or give a counterexample: if a and b are both irrational, then a + b is irrational.

12. Prove or give a counterexample: If a^2 is irrational, then a is irrational.