Mathematics 300

Quiz 25 **Name:**______________________________

You must show your work to get full credit.

Proposition. All natural numbers are even.

Proof We will prove if a finite set, S, of natural contains an even number then all the elements of the set are even. As every natural number is contained in some finite set, this will show all natural numbers are even. We will use induction on $n = \#(S)$, the number of elements in S.

Base case: $n = 1$. Then S is a set with at least one even element. As S only has one element, we have that all elements of S are even. Therefore the base case holds.

Induction step: Assume we know the result is true for sets of size n and let $\#(S) = n + 1$. We are assuming that S has at least one even element, say $k \in S$ is even. Then split S into two set $S_1 = \{k\}$ and $S_2 = S \setminus \{k\}$ (that is S_2 is S with k removed.) Applying the induction hypothesis to S_2 (which has n elements) we find all elements of S_2 are even. Therefore all elements of $S = S_1 \cup S_2$ are even which finishes the proof. $\square$

1. What is the error in this proof?