Mathematics 300

Quiz 26 **Name:**_____

You must show your work to get full credit.

Define a sequence recursively by

$$a_{n+1} = \sqrt{2 + a_n}, \qquad a_0 = \sqrt{2}.$$

Then

$$a_1 = \sqrt{2 + \sqrt{2}}$$

$$a_2 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

$$a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$$

$$a_4 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}$$

1. Prove $a_n < 2$ for all $n \in \mathbb{N}$.

2. Prove for $a_{n+1} > a_n$ for $n \in \mathbb{N}$.