Mathematics 300

Quiz 30 **Name:**_____

You must show your work to get full credit.

1. Use induction to show the n-derivative of

$$f(x) = \frac{1}{x}$$

is

$$f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}.$$

2. (a) Use induction to show: if $a \equiv b \pmod{m}$, then $a^n \equiv b^n \pmod{m}$ for all $n = 1, 2, 3, \ldots$.

(b) Show that $7|(9^n - 2^n)$ for $n = 1, 2, 3, \ldots$.

3. Show that if McNuggets come in packages of size 4 and 6, then for any it is possible to buy exactly m McNuggets for any even number $m \geq 4$.

4. Let a_n be defined by the recursion

$$a_{n+1} = \frac{2}{3}a_n + 6, \qquad a_1 = 3.$$

(a) Compute $a_2 =$ _____ $a_3 =$ _____ $a_4 =$ _____

(b) Prove $a_n < 18$ for all n.

(c) Prove $a_{n+1} > a_n$ for all $n \geq 1$.

5. Use Venn diagrams to show each of the following:
(a) $(A \cap B)^c = A^c \cup B^c$.

(b) $A \cap (B \cup B) = (A \cap B) \cup (A \cap C)$.

6. Draw Venn diagrams showing the following relations between A, B, and C.
(a) $A \subseteq B$ and $B \cap C = \varnothing$.

(b) $A \cap C = B \cap C$.

7. Let

$$A = \{n \in \mathbb{Z} : nis \text{ even } \}$$
$$B = \{x(x+1) : x \in\}.$$

(a) Show $B \subseteq A$.
(b) Show A is not a subset of B.

8. Let

$$A = \{k \in \mathbb{Z} : k \equiv 1 \pmod 3\}$$
$$B = \{9x + 6y + 4 : x, y \in \mathbb{Z}\}.$$

Prove $A = B$.