Mathematics 552 Test #1 Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (30 Points) Compute the following:

   (a) $(4-3i)(6+2\,i)$ _____

   (b) $\dfrac{5+2\,i}{3+4\,i}$ _____

   (c) $\mathrm{Arg}(-3-3\,i)$ _____

   (d) $\arg(-1+\sqrt{3}\,i)$ _____

   (e) $(-1+\sqrt{3}\,i)^{11}$ _____

   (f) $\left|\dfrac{(\sqrt{3}+i)^9}{(\sqrt{3}-i)^8}\right|$ _____

   (g) $e^{2+\frac{\pi}{4}\,i}$ _____

   (h) $\mathrm{Im}[(4-5\,i)(x+y\,i)]$ _____

2. (10 Points) Find all values of $(-27)^{\frac{2}{3}}$.

3. (10 Points) Show that $e^z + e^{\overline{z}} = 2e^{\operatorname{Re} z} \cos(\operatorname{Im} z)$

4. (15 Points) Draw pictures of the following sets of complex numbers:
   (a) $|z - 4 - 3\,i| < 2$

   (b) $2 < |z| < 3$ and $\dfrac{\pi}{2} < \mathrm{Arg}(z) < \pi$

   (c) $\mathrm{Re}[(2 - 3\,i)z] < 6$

5. (10 Points) Let

$$A = \{z : 1 < |z| < 3\}, \qquad B = \{z : |z| \geq 5\}, \qquad C = \{z : \operatorname{Re} z > 0, |z| \leq 4\}$$

Then

(a) Which of these sets is open? _____

(b) Which of these sets are domains? _____

(c) Which of these sets are regions? _____

(d) Which of these sets are bounded? _____

6. (15 Points) Solve the following equations:

(a) $\dfrac{1+z}{1-z} = 3 + 4\,i$ $\qquad z =$ _____

(b) $z^2 + (-3 + 3\,i)z - 5\,i = 0$ $\qquad z =$ _____

7. (10 Points) One cube root of -512 is $4 + 4\sqrt{3}\,i$. Plot all of the cube roots.