Mathematics 122

Quiz #24 Name: Key

You must show your work to get full credit.

(1) Let a be a constant and set $f(x) = x^3 - 6ax^2 + a^4$.

2 pts (a) Then where is the graph of $y = f(x)$ concave up?

$f'(x) = 3x^2 - 12ax$

$f''(x) = 6x - 12a = 6(x - 2a)$ $\quad x > 2a$

$f(x)$ is concave up when $f''(x) = 6(x-2a) > 0$

(b) What is the inflection point of $y = f(x)$ (give both the x and y coordinates)?

1 pt The graph changes concavity at $x = 2a$ $\quad x = 2a,\ y = -64a^3 + a^4$

$y = f(2a) = (2a)^3 - 6(2a)^2 + a^4$

$= 8a^3 - 72a^3 + a^4 = -64a^3 + a^4$

2 pts (2) Let b be a constant. What it the tangent line to $y = 2bx^2$ where $x = -1$?

$y - y_0 = m(x - x_0)$ $\quad y - 2b = -4b(x+1)$

$x_0 = -1$ $\quad$ or $\quad y = -4bx - 2b$

$y_0 = 2b(-1)^2 = 2b$

$y' = 4bx$

$m = y'(-1) = 4b(-1) = -4b$

$y - 2b = -4b(x - (-1))$