Mathematics 241 Test #3 Name: Key

Show your work to get credit. An answer with no work will not get credit.

1. (10 points) Find the critical point(s) of $f(x, y) = x^2 + 2xy + 2y^2 + 4x - 6y$ and determine they a local maximum, minimum, or saddle point.

$f_x = 2x + 2y + 4 = 0$

$f_y = 2x + 4y - 6 = 0$

so

$0 = f_y - f_x = 2y - 10 = 0$

$2y = 10$

$y = 5$

so $2x + 2(5) + 4 = 0$

$2x = -14$

$x = -7$

Critical points are: $(-7, 5)$

Which are local maximums: none

Which are local minimums: $(-7, 5)$

Which are saddle points: none

$f_{xx} = 2,\ f_{xy} = 2,\ f_{yy} = 4$

$D = f_{xx} f_{yy} - f_{xy}^2 = 2 \cdot 4 - 2^2 = 4 > 0 \quad f_{xx} = 2 > 0$

so all critical points are local mins

2. (10 points) Find the maximum of $f(x, y) = 6x - 8y$ on the circle $x^2 + y^2 = 25$.

Use Lagrange multiplier

The maximum value is: 50

It occurs at the point: $(3, 4)$

$f = 6x - 8y$

$g = x^2 + y^2$

$f_x = \lambda g_x \qquad 6 = \lambda(2x) \qquad x = \frac{3}{\lambda}$

$f_y = \lambda g_y \qquad -8 = \lambda(2y) \qquad y = \frac{-4}{\lambda}$

Use this in $x^2 + y^2 = 25$

$\left(\frac{3}{\lambda}\right)^2 + \left(\frac{-4}{\lambda}\right)^2 = 25$

$\frac{25}{\lambda^2} = 25$

$\lambda^2 = 1$

$\lambda = \pm 1$

so critical points are

$\lambda = 1 \quad (x, y) = (3, -4)$

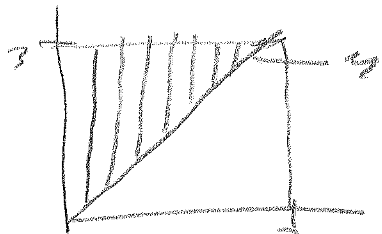
$f(3, -4) = 3(6) - 8(-4)$
$= 18 + 32 = 50$

$\lambda = -1 \quad (x, y) = (-3, 4)$

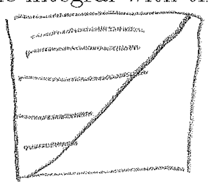
$f(-3, -4) = -50$

3. (15 points) For the integral $\int_0^3 \int_x^3 f(x,y)\,dy\,dx$

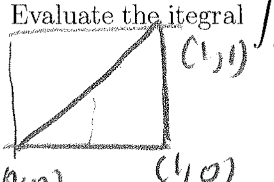
(a) Draw the region of integration.



(b) Write the integral with the order of intergration reversed. $\int_0^3 \int_0^y f(x,y)\,dx\,dy$



4. (10 points) Evaluate the itegral $\iint_D 28x^2y^3\,dA$ where D is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$.



The integral is 1

$$\int_0^1\int_0^x 28x^2y^3\,dy\,dx = \int_0^1 7x^2y^4\Big|_{y=0}^{x}dx$$

$$= \int_0^1 7x^6\,dx = x^7\Big|_0^1 = 1$$

5. (10 points) Evaluate the integral $\int_0^1 \int_0^x \int_0^{xy} 24z\,dz\,dy\,dx$

The integral is: $\frac{2}{3}$

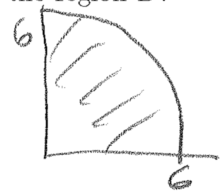
$$= \int_0^1\int_0^x 12z^2\Big|_{z=0}^{xy} dy\,dx$$

$$= \int_0^1\int_0^x 12x^2y^2\,dy\,dx$$

$$= \int_0^1 4x^2y^3\Big|_{y=0}^{x} dx = \int_0^1 4x^5\,dx = \frac{4x^6}{6}\Big|_0^1 = \frac{4}{6} = \frac{2}{3}$$

6. (15 points) Let D be the region define by $x^2 + y^2 \leq 36$, $x \geq 0$ and $y \geq 0$.

(a) Draw the region D.



(b) Use polor coordinates to compute $\iint_D (x^2 + y^2)\, dA$ $\underline{\quad 162\pi \quad}$

$$= \int_0^{\pi/2} \int_0^6 r^2 \, r\, dr\, d\theta$$

$$= \frac{\pi}{2} \int_0^6 r^3 dr = \frac{\pi}{2} \left. \frac{r^4}{4} \right|_0^6 = \frac{\pi (6)^4}{8} = 162\pi$$

7. (10 points) Let C be the curve (t^2, t^3) with $0 \leq t \leq 1$. Evaluate the line integral

$$\int_C y\, dx + 2x\, dy.$$

$\underline{\quad \frac{8}{5} \quad}$

On this curve

$x = t^2 \quad dx = 2t\, dt$

$y = t^3 \quad dy = 3t^2 dt$

$$\int_C y\,dx + 2x\,dy = \int_0^1 (t^3)(2t\,dt) + (2t^2)(3t^2)\,dt$$

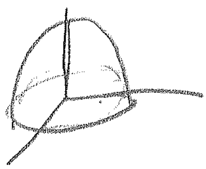
$$= \int_0^1 (2t^4 + 6t^4)\,dt = \int_0^1 7t^4\,dt$$

$$= \frac{8}{5} t^5 \Big|_0^1 = \frac{8}{5}$$

8. (20 points) Let D be the region of $\mathbf{R}^3$ that is above the x-y plane, and below the graph of $z = 4 - x^2 - y^2$.

(a) Set up, but do not evalute, the integral for the volume of D in x, y, z.

$z = 4 - x^2 - y^2$



$$\iint_{4-x^2-y^2 \geq 0} \int_0^{4-x^2-y^2} 1\, dz\, dx dy$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4+x^2}} \int_0^{4-x^2-y^2} 1\, dz\, dy dx$$

(b) Set up, but do not evalute, the integral for the volume of D in cylindrical coordinates.

$$\int_0^{2\pi} \int_0^{2} \int_0^{4-r^2} 1\, r\, dz\, dr\, d\theta$$

(c) Use your answer of part (b) to find the volume of D.

The volume is 8π

$$= 2\pi \int_0^2 \int_0^{4-r^2} r\, dz\, dr$$

$$= 2\pi \int_0^2 r(4-r^2)\, dr$$

$$= 2\pi \int_0^2 (4r - r^3)\, dr$$

$$= 2\pi \left(\frac{4r^2}{2} - \frac{r^4}{4}\right)\Big|_0^2$$

$$= 2\pi \left(\frac{4(2)^2}{2} - \frac{2^4}{4}\right)$$

$$= 2\pi(8-4) = 8\pi$$