Mathematics 552 Homework due Friday, January 13

(1) Compute the following:
   (a) $(1+3i)(4-2i)$
   (b) $\dfrac{3+2i}{4-i}$
   (c) z^3 where $z = \dfrac{-1+\sqrt{3}\,i}{2}$.

(2) Let $z = 1+i$.
   (a) Find the polar form $z = re^{i\theta}$ of z.
   (b) Compute z^{15}.

(3) Let $z = 1-\sqrt{3}\,i$.
   (a) Find the polar form $z = re^{i\theta}$ of z.
   (b) Compute z^{10}.

(4) Show that for any integer n that $e^{2\pi ni} = 1$ and therefore $e^{(\theta+2\pi n)i} = e^{\theta i}$.

(5) Let n be a positive integer and k any integer. Let

$$\omega = e^{\frac{1}{n}(2k\pi i)}.$$

Show that

$$\omega^n = 1.$$

Thus ω is an n-th root of unity (i.e. an n-th root of 1.)