Mathematics 122

Quiz #14 Name: Key

You must show your work to get full credit.

1. Let a, b, c be constants. Compute the following derivatives.

(a) $f(x) = 5\sqrt{x} - \frac{8}{x^3} = 5x^{\frac{1}{2}} - 8x^{-3}$ $f'(x) = \underline{\frac{5}{2}x^{-\frac{1}{2}} + 24x^{-4}}$

$f'(x) = \frac{1}{2}5x^{\frac{1}{2}-1} - (-3)x^{-3-1}$

(b) $w = az^3 + ab^2z$ $w' = \underline{3az^3 + ab^2}$

note: $(ab^2z)' = ab^2$
for the same reason $(17z)' = 17$

(c) $C = \frac{4bc^3}{\sqrt{q}} + 4b^5 = 4bc^3q^{-\frac{1}{2}} + 4b^2$ $\frac{dC}{dq} = \underline{-2bc^3q^{-3/2}}$

$\frac{dc}{dq} = -\frac{1}{2}4bc^3q^{-\frac{1}{2}-1} + 0$
$= -2bc^3q^{-\frac{3}{2}}$ (note $4b^2$ is constant so $(4b^2)' = 0$)

2. Find the equation of the tangent line to $y = x - x^3$ at the point where $x = 2$.

The equation is $\underline{y = -6 - 11(x-2) \text{ or } y = -11x + 16}$

The equation of the tangent line to $y = f(x)$ where $x = a$ is

$y = f(a) + f'(a)(x-a)$

In our case $f(x) = x - x^3$, $f'(x) = 1 - 3x^2$
$a = 2$ so
$f(a) = f(2) = 2 - 2^3 = -6$
$f'(a) = f'(2) = 1 - 3(2)^2 = 1 - 12 = -11$
so the tangent line is

$y = -6 + (-11)(x-2)$
$= -6 - 11(x-2)$

(or $y = -6 - 11x + 22 = -11x + 16$)