Mathematics 122

Quiz #15 Name: Key

You must show your work to get full credit.

We need some review on a couple of things. First if a, b, c are constants then any expression just evolving a, b, c is also a constant and thus has zero derivative. For example if

$$y = 3x^2 + 4b^3 \qquad \text{then} \qquad y = 6x + 0 = 6x$$

as $4b^3$ is a constant and thus $(4b^3)' = 0$. Likewise if

$$y = 2ac^3x^4 \qquad \text{then} \qquad y' = 4(2ac^3)x^3 = 8ac^3x^3$$

for the same reason that if

$$y = 7x^4 \qquad \text{then} \qquad y' = 4(7)x^3 = 28x^3.$$

Second, the equation of the tangent line to curve $y = f(x)$ at the point where $x = a$ is

$$y = f(a) + f'(a)(x - a).$$

(Note this is the same as the equation for the linear approximation we know and love from the last couple of weeks.)

As an example: find the tangent line to $y = x^2 - x$ at the point where $x = 3$. Then $a = 3$, $f(x) = x^2 - x$ and so $f'(x) = 2x - 1$. Therefore

$$f(a) = f(3) = 3^2 - 3 = 9 - 3 = 6 \qquad \text{and} \qquad f'(a) = f'(3) = 2(3) - 1 = 5.$$

Thus the equation of the tangent line is

$$y = 6 + 5(x - 3).$$

1. Let a, b, c be constants. Find the following derivatives

(a) $y = 5e^x$. $y' =$ $5e^x$

(b) $C = ace^q - 9b^3$. $\frac{dC}{dq} =$ ace^q

$(ace^q)' = ace^q$

$(9b^3)' = 0$

2. What is the tangent line to $y = x^2 - x$ at the point where $x = -1$?

The equation is $y = 2 - 3(x+1)$

or $y = -3x - 1$

$f(x) = x^2 - x$

$f'(x) = 2x - 1$

$a = -1$

$f(a) = f(-1) = (-1)^2 - (-1) = 2$

$f'(a) = 2(-1) - 1 = -3$

so eqn is $y = 2 + (-3)(x - (-1)) = 2 - 3(x+1)$

$= 2 - 3x - 3$

$= -3x - 1$