Mathematics 122 Test #1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) The variables p and q are related as in the table

$\Delta p =$ .5 .5 .5

p	3.0	3.5	4.0	4.5
q	8.5	7.0	5.5	4.0

$\Delta q =$ -1.5 -1.5 -1.5

(a) Explain why the relation between p and q could be linear. (This will involve both doing some calculations and writing at least one sentence explaining why the calculations are relevant.)

The slopes are a constant $\frac{\Delta q}{\Delta p} = \frac{-1.5}{.5} = -3$.
Constant slope implies it is a straight line.

(b) Find q as a function of p.

$q = -3p + 17.5$

$\frac{\Delta q}{\Delta p} = \frac{q-8.5}{p-3.0} = -3$

$q - 8.5 = -3(p-3.0)$ → $q = -3p + 9 + 8.5 = -3p + 17.5$

(c) What is the value of p when $q = 5$?

$4.166666\overline{6}$

$5 = -3p + 17.5$

$3p = 17.5 - 5 = 12.5$

$p = \frac{12.5}{3} = 4.1666\ldots$

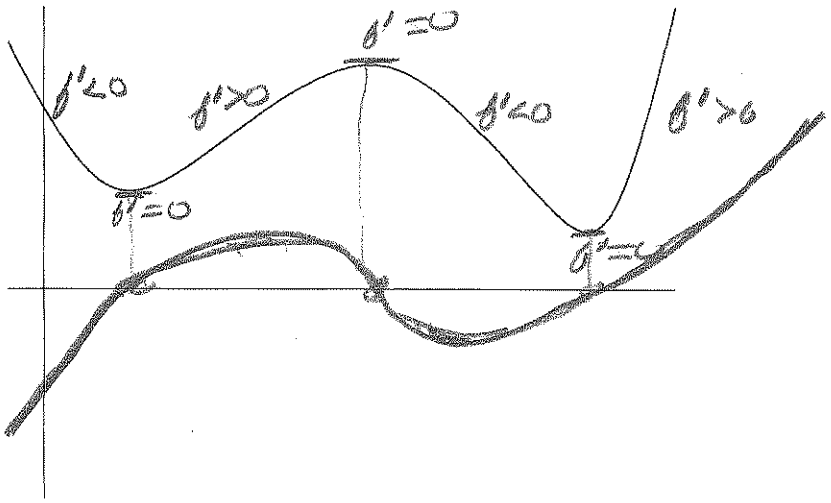
2. (5 points) Use your calculator to compute $f'(-1.4)$ when $f(x) = \dfrac{3 - x^2}{2^{1+x} + 4}$

$f'(-1.4) =$ .56436

Write what you punched into the calculator.

nDriv((3-X^2)/(2^(1+X)+4), X, -1.4)

3. (8 points) The is the graph of a function $y = f(x)$. Draw the graph of the graph of the derivative function $y = f'(x)$ on the same axis.



4. (10 points) You invest \$3,000 at 6% interest compounded annually.

(a) Give a formula for the principal, $P(t)$, after t years.

$P(t) =$ $3000(1.06)^t$

(b) How much is in the account is there after ten years?

$P(10) = 3000\ (1.06)^{10}$
$= 5372.54$

Amount after ten years = \$5372.54

(c) How long does it take for the investment to reach \$10,000?

Number of years to reach \$10,000 = 20.66 yrs.

We need to solve

$3000\ (1.06)^t = 10000$

$(1.06)^t = 10000/3000$

$t \ln(1.06)^t = \ln(10000/3000)$

$t = \ln(10000/3000)/\ln(1.06)$
$= 20.66$

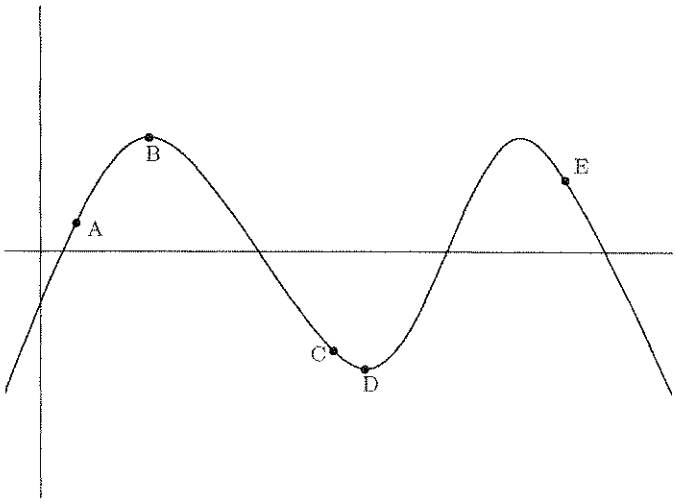
5. (5 points) The height, H, of a tree in feet is a function of its age, t, in years. $H = f(t)$. If $f(10) = 15$ and $f'(10) = 2$

What are the units of 10? years

What are the units of 15? feet

What are the units of 2? feet/year

6. (8 Points) Let $y = f(x)$ have the following graph.



At which of the labeled points is f negative? C, D
(i.e. below axis)

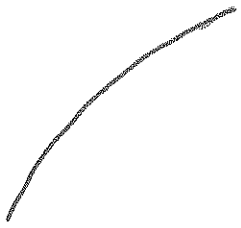
At which of the labeled points is f' zero? B, D
(i.e. horizontal tangent)

At which of the labeled points is f' positive ? A
(i.e. increasing)

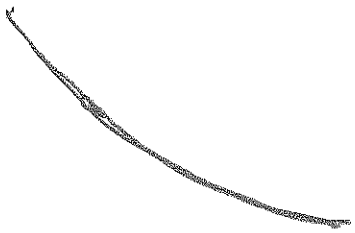
At which of the labeled points if f'' positive? C, D
(i.e. concave up)

7. (10 points) Draw graphs of functions, f, with the following properties.

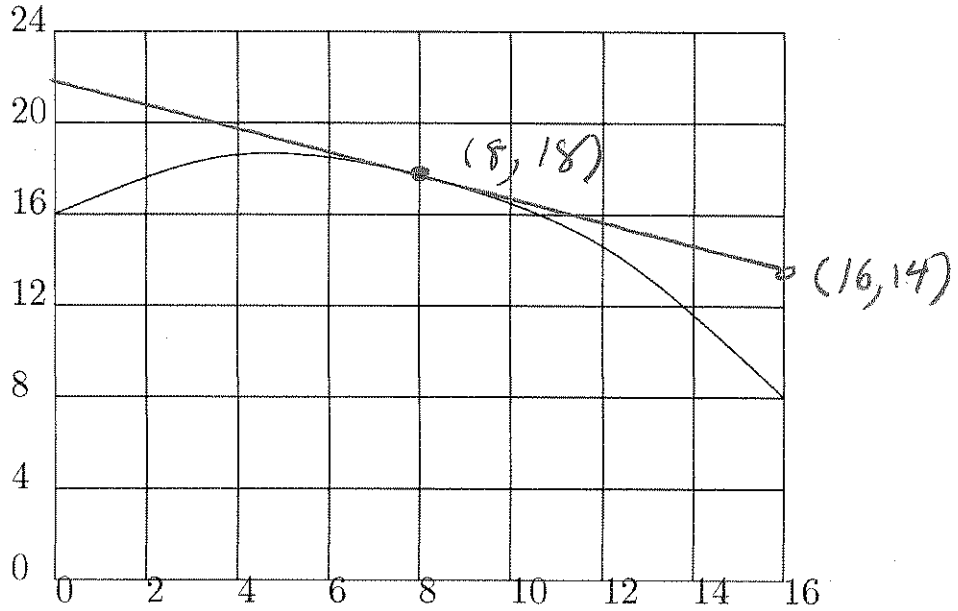
(a) f is increasing at a decreasing rate.



(b) $f' < 0$ and $f'' > 0$



8. (10 points) Let $y = f(x)$ have the following graph:



(a) Draw the tangent line at the point where $x = 8$ and label, on the graph, two points on this tangent line showing both the x and y coordinates.

(b) Estimate the derivative $f'(8)$.

$f'(8)$ = slope of tangent line $f'(8) \approx -.5$

$= \frac{\Delta y}{\Delta x} = \frac{14-18}{16-8} = \frac{-4}{+8} = -\frac{1}{2}$

(of course other answers were counted as correct)

9. (8 points) The following table gives some values for $y = f(x)$.

$\Delta x =$ .2 .2 .2

x	1.0	1.2	1.4	1.6
$f(x)$	9.8	9.4	8.8	8.0

$\Delta y = \Delta f =$ -.4 -.6 -.8

Make a table for the derivative.

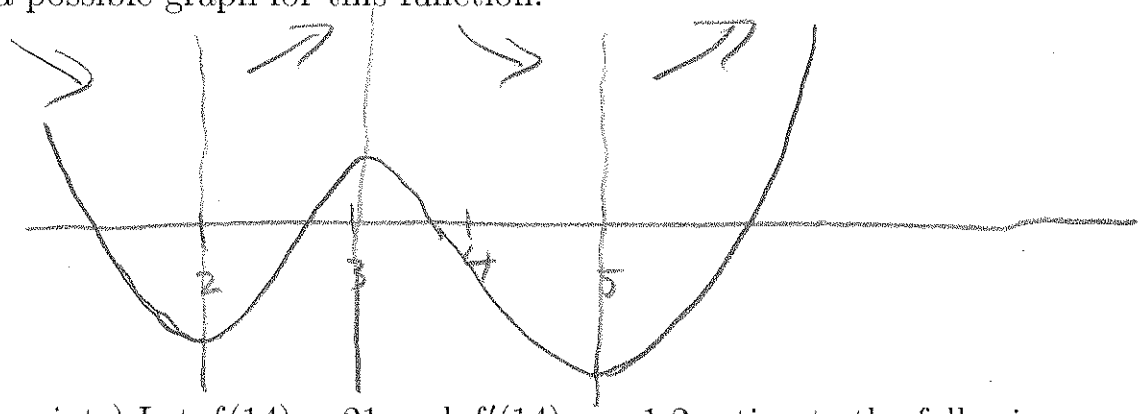
x	1.1	1.3	1.5
$f'(x)$	-2	-3	-4

$\frac{\Delta y}{\Delta x} =$ $\frac{-.4}{.2}$ $\frac{-.6}{.2}$ $\frac{-.8}{.2}$

10. (8 points) Let f be a function that satisfies

- $f'(x) < 0$ for $x < 2$ and $3 < x < 5$ (is decreasing)
- $f'(x) > 0$ for $2 < x < 3$ and $5 < x$. (is increasing)

Draw a possible graph for this function.



11. (8 points) Let $f(14) = 21$ and $f'(14) = -1.2$ estimate the following

The basic linear approximation formula is

$f(x) \approx f(a) + f'(a)(x-a)$

$f(14.2) \approx$ 20.76

$f(13.7) \approx$ 21.36

$f(14.2) \approx 21 + (-1.2)(.2) = 20.76$

$f(13.9) \approx 21 + (-1.2)(-.3) = 21.36$

12. (10 points) A winery produces a Merlot wine. It costs them $C(q)$ to produce q bottles of the wine and selling q bottles brings in a revenue of $R(q)$ dollars. The cost and revenue of producing $q = 500$ bottles is $C(500) = \$4{,}000$ and $R(500) = \$8{,}000$. The marginal cost and revenue are $MC(500) = 12$ and $MR(500) = 14$.

(a) What are the units of $MC(500) = 12$?

Units are: $/bottle

(b) What is the profit in producing the 501-st bottle of wine?

$MR(500) - MC(500)$

$= 14 - 12 = 2$

Profit is $2.00

(c) If the winery is producing 500 bottles of Merlot, should they increase or decrease production? Explain your answer.

They should increase production, as producing another bottle brings in a profit of $2.00