Problem 1. Let $\lim_{n\to\infty} a_n = A$. Show

$$\lim_{n\to\infty} \frac{a_n + a_{n+1} + a_{n+2}}{3} = A.$$

□

Problem 2. Let $w(x)$ be a positive integrable function on $[a, b]$. Then for any integrable function f on $[a, b]$ define the ***weighted average*** of f on $[a, b]$ to be

$$A_w(f) := \frac{\displaystyle\int_a^b f(x)w(x)\,dx}{\displaystyle\int_a^b w(x)\,dx}.$$

Show that if f is continuous on $[a, b]$ that there is a point $\xi \in [a, b]$ such that

$$f(\xi) = A_w(f).$$

□

Problem 3. Give an example to show that the last problem is false if f is not continuous. □