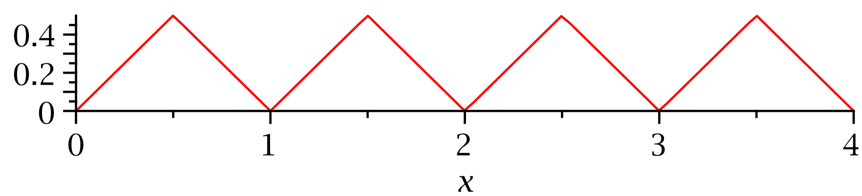
Graph of a continuous nowhere differentiable function.

Here are some pictures of the nowhere differentiable function we defined in class. First let

$$\varphi(x) = \text{distance of } x \text{ to the nearest integer.}$$

This has periodic (i.e. $\varphi(x+1) = \varphi(x)$) and a saw toothed graph



Then $f(x)$ is defined by the infinite sum

$$f(x) = \sum_{k=0}^{\infty} \frac{\varphi(2^k x)}{2^k} = \varphi(x) + \frac{\varphi(2x)}{2} + \frac{\varphi(2^2 x)}{2^2} + \frac{\varphi(2^3 x)}{2^3} + \cdots$$

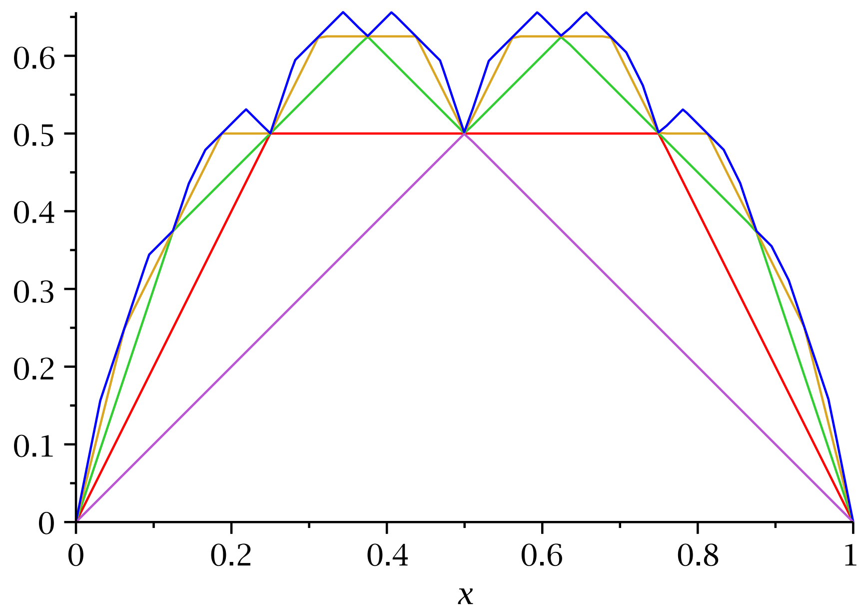
This converges uniformly and therefore is continuous. Let

$$S_n(x) = \sum_{k=0}^{n} \frac{\varphi(2^k x)}{2^k}$$

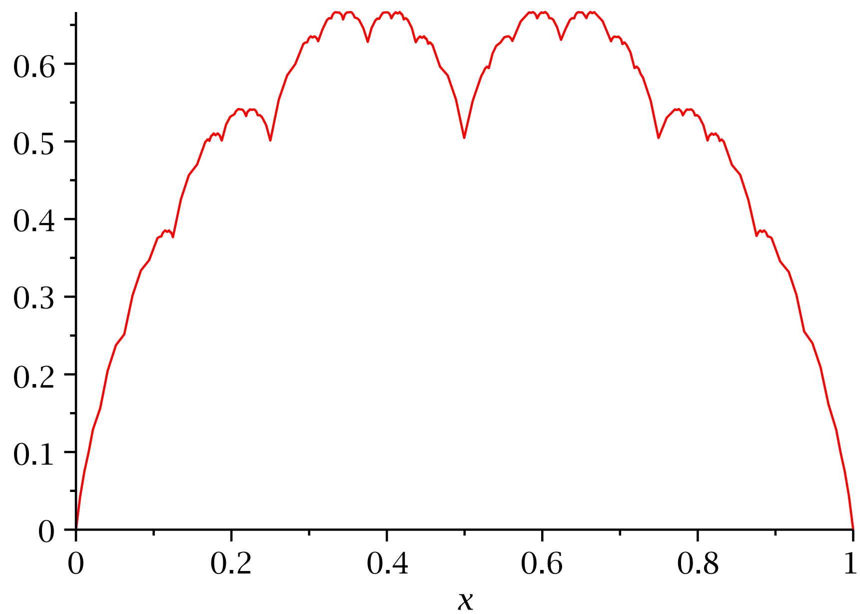
be the sum of the first $(n+1)$ terms.

$$S_0(x) = \varphi(x), \quad S_1(x) = \varphi(x) + \frac{\varphi(2x)}{2}, \quad S_2(x) = \varphi(x) + \frac{\varphi(2x)}{2} + \frac{\varphi(2^2x)}{2^2} + \frac{\varphi(2^3x)}{2^3}$$

and so on. Here is are the graphs of $S_0, S_1, \ldots, S_4$ on $[0,1]$.



Finally here is the graph of f on $[0, 1]$ (or at least what should the graph to an accuracy smaller than a size of a pixel, but it does not look that detailed to me).



This has a property like that of a fractal, that no matter how much you zoom in on it, there will still be detail in the form of local maximums and minimums.