Mathematics 122 Test #1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (15 points) The variables p and q are related as in the table

p	10.0	10.5 ①	11.0 ②	11.5 ③
q	9.5	8.0	6.5	5.0

(a) Explain why the relation between p and q could be linear. (This will involve both doing some calculations and writing at least one sentence explaining why the calculations are relevant.)

At ① $\frac{\Delta q}{\Delta p} = \frac{8-9.5}{10.5-10} = \frac{-1.5}{.5} = -3$

At ② $\frac{\Delta q}{\Delta p} = \frac{6.5-8}{11.0-10.5} = \frac{-1.5}{.5} = -3$

At ③ $\frac{\Delta q}{\Delta p} = \frac{5.0-6.5}{11.5-11} = \frac{-1.5}{.5} = -3$

It is linear because the slopes $\frac{\Delta q}{\Delta p}$ are constant.

(b) Find q as a function of p. $q = 9.5 - 3(p-10)$ or $q = -3p + 39.5$

$$\frac{\Delta q}{\Delta p} = \frac{q-9.5}{p-10} = -3$$

$$q - 9.5 = -3(p-10)$$

$$q = 9.5 - 3(p-10)$$

$$= 9.5 - 3p + 30$$

$$= -3p + 39.5$$

(c) What is the value of q when $p = 10.7$? 7.4

Let $p = 10.7$ in part (b)

$$q = 9.5 - 3(10.7-10)$$

$$= 9.5 - 3(.7)$$

$$= 9.5 - 2.1$$

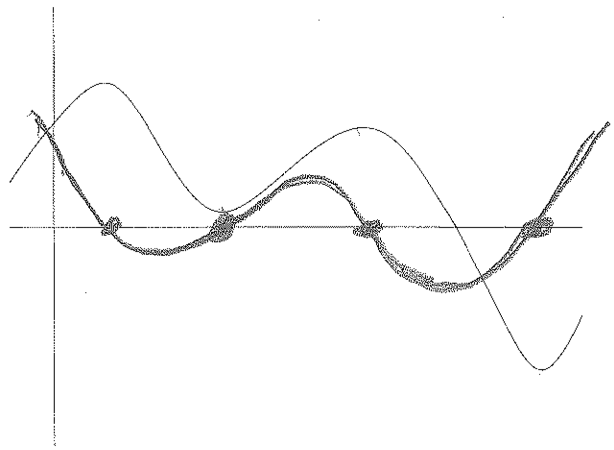
$$= 7.4$$

2. (7 points) Use your calculator to compute $f'(-1.5)$ when $f(x) = \dfrac{x^2-3}{2^{x+1}+5}$

$f'(-1.5) =$ $-.514374$

Write what you punched into the calculator. nDeriv((X^2-3)/(2^(X+1)+5),X,-1.5)

3. (10 points) On the same axis draw the graph of the derivative $y = f'(x)$.



4. (10 points) You invest \$5,000 at 7% interest compounded continuously.

(a) Give a formula for the principal $P(t)$ after t years.

We know that when compounded continuously principle grows at $P(t) = P_0 e^{rt}$

P_0 = initial amount, r = rate

$P(t) =$ $5000 e^{.07t}$

(b) How much is in the account is there after fifteen years?

Just plug in $t = 15$

Amount after fifteen years = \$14,288.26

$P(15) = 5000 e^{.07(15)} =$

(c) How long does it take for the investment to reach \$35,000?

Number of years to reach \$35,000 = 27.80 years.

we need to solve

$P(t) = 5000\ e^{.07t} = 35000$

$e^{.07t} = 35000/5000$

$.07t = \ln(35000/5000)$

$t = \ln(35000/5000)/.07$

5. (10 points) The half life of a substance is 2,000 years. A sample is found where 12% of the original substance is left. How old is the sample?

$P(t) = P_0 (a)^t$. we know

$P(2000) = P_0 (a)^{2000} = \frac{1}{2} P_0$

i.e. $a^{2000} = .5$

$a = (.5)^{\frac{1}{2000}}$

so $P(t) = P_0 (.5)^{\frac{t}{2000}}$

The age is 6117.8 years.

Now solve

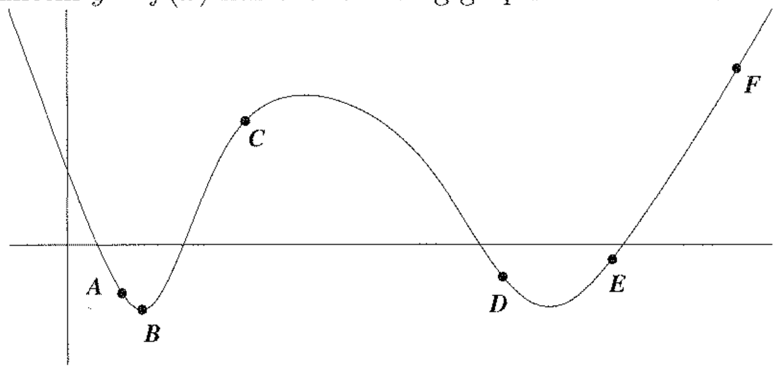
$P(t) = P_0 (.5)^{\frac{t}{2000}} = .12 P_0$

$(.5)^{\frac{t}{2000}} = .12$

$\frac{t}{2000} \ln(.5) = \ln(.12)$

$t = \frac{2000(\ln(.12))}{\ln(.5)} =$

6. (10 points) The functin $y = f(x)$ has the following graph.



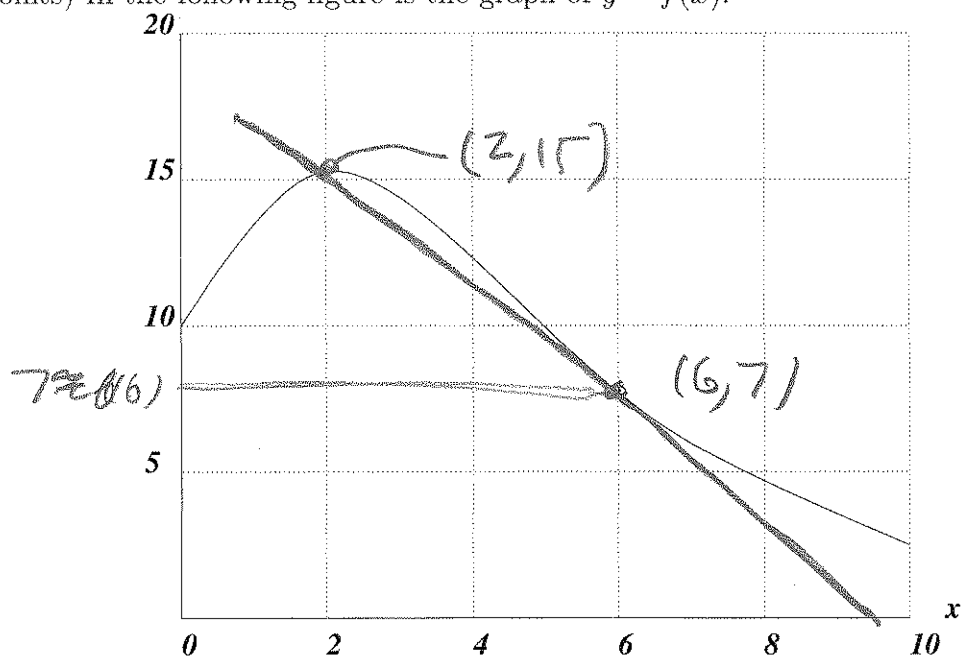
At which of the labeled points is $f > 0$? C, F

At which of the labeled points is $f' > 0$? C, E, F

At which of the labeled points is $f'' > 0$ A, B, D, E, F

At which of the labeled points is the function concave down? C

7. (10 points) In the following figure is the graph of $y = f(x)$.



What is the value of $f(6)$? ≈ 7

What is the average rate of change between $x = 2$ and $x = 8$? ≈ -1.83

$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(2)}{8-2} \approx \frac{4.5 - 15.5}{8-2} = -1.83$$

Draw the tangent line to the graph at the point where $x = 6$, label two points on that line and use those points to estimate the derivative $f'(6)$.

$$f'(6) \approx \frac{\Delta y}{\Delta x} = \frac{f(6) - f(2)}{6-2} = \frac{7-15}{6-2} = -2$$

$f'(6) \approx$ $-2.$

8. (10 points) The cost, C, in dollars of producing apple cider is a function of the number, ℓ, of liters produced. That is $C = f(\ell)$. If

$$f(200) = 56, \qquad f'(200) = .32$$

What are the units of 200? liters

What are the units of 56? dollars

What are the units of .32? dollars/litter

Use these numbers to estimate $f(201.5)$.

$f(201.5) \approx$ \$56.48

$$f(201.5) \approx f(200) + f'(200)(201.5 - 200)$$
$$= 56 + (.32)(1.5)$$
$$= 56.48$$

9. (10 points) The following table gives some values for $y = f(x)$.

x	1.0	1.2	1.4	1.6
$f(x)$	5.2	4.9	4.6	4.0

(a) What is the average rate of change between $x = 1.2$ and $x = 1.6$?

Average rate of change is -2.25

$$\frac{\Delta f}{\Delta x} = \frac{f(1.6) - f(1.2)}{1.6 - 1.2} = \frac{4.0 - 4.9}{.4} = \frac{-.9}{.4} = -\frac{9}{4}$$

(b) Estimate $f'(1.3) \approx \dfrac{f(1.4) - f(1.2)}{1.4 - 1.2}$

$f'(1.3) \approx$ -1.5

$$= \frac{4.6 - 4.9}{.2} = \frac{-.3}{.2} = -\frac{3}{2} = -1.5$$

10. (8 points) Draw the graph of a function with $f < 0$, $f' > 0$ and $f'' < 0$.

$f < 0 \Rightarrow$ below x-axis

$f' > 0 \Rightarrow$ increasing

$f'' < 0 \Rightarrow$ concave down