Mathematics 122 Test #1 Name: Key

You are to use your own calculator, no sharing.

Show your work to get credit.

1. (50 points) Compute the following derivatives. Here a, b and c are constants.

(a) $y = 3x^4 - 4x^2 + 6x - 2$. $y' =$ $12x^3 - 8x + 6$

(b) $f(t) = \sqrt{t} = t^{\frac{1}{2}}$

$f'(t) = \frac{1}{2} t^{-\frac{1}{2}}$

$f'(t) =$ $\frac{1}{2} t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$

(c) $w = \frac{6}{z^4} = 6z^{-4}$

$\frac{dw}{dz} = -24z^{-5}$

$\frac{dw}{dz} =$ $-24z^{-5} = \frac{-24}{z^5}$

(d) $g(x) = 3ax^3 + 2b^2x + b^4$. ← constant

$g'(x) = 9ax^2 + 2b^2 + 0$

$g'(x) =$ $9ax^2 + 2b^2$

(e) $y = e^x + e^b$

$y' = e^x + 0$ (as e^b is constant)

$\frac{dy}{dx} =$ e^x

(f) $P(t) = 2000e^{.05t}$.

$P'(t) = 2000e^{.05t}(.05)$

$= 100e^{.05t}$

$P'(t) =$ $100e^{.05t}$

(g) $P(t) = 2000e^{at}$. $P'(t) =$ $2000a\,e^{at}$

(h) $f(x) = x\ln(x) - x$.

$f'(x) = 1\ln(x) + x\left(\frac{1}{x}\right) - 1$

$= \ln(x) + 1 - 1$

$= \ln(x)$

$f'(x) =$ $\ln(x)$

(i) $y = 4(x^2 + x)^5$.

$y' = 5(4)(x^2+x)^4(2x+1)$

$y' =$ $20(x^2+x)^4(2x+1)$

(j) $C(q) = 100e^{q+q^2}$.

$C'(q) = 100e^{q+q^2}(1+2q)$

$C'(q) =$ $100(1+2q)e^{q+q^2}$

(k) $h(x) = \sqrt{e^x + 1} = (e^x+1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(e^x+1)^{-\frac{1}{2}}(e^x+0)$

$= \frac{1}{2}(e^x+1)^{-\frac{1}{2}}e^x$

$h'(x) =$ $\frac{1}{2}(e^x+1)^{-\frac{1}{2}}e^x$

$= \frac{e^x}{2\sqrt{e^x+1}}$

(l) $y = 3x^5e^x$.

$y' = 15x^4e^x + 3x^5e^x$

$= (15x^4 + 3x^5)e^x$

$\frac{dy}{dx} =$ $(15x^4+3x^5)e^x$

(m) $h(u) = 2u^5(u^2 - u)^3$

$h'(u) = 10u^4(u^2-u)^3 + 2u^5(3)(u^2-u)^2(2u-1)$

$h'(u) =$ $10u^4(u^2-u)^3 + 6u^5(u^2-u)^2(2u-1)$

(n) $y = \frac{x^3}{e^{2x}} = x^3e^{-2x}$

$y' = 3x^2e^{-2x} + x^3(e^{-2x}(-2))$

$= (3x^2-2x^3)e^{-2x}$

$y' =$ $(3x^2-2x^3)e^{-2x}$

(o) $y = \frac{2x}{x+3}$

$y' = \frac{2(x+3)-2x(1)}{(x+3)^2} = \frac{2x+6-2x}{(x+3)^2}$

$y' =$ $\frac{6}{(x+3)^2}$

(p) $y = \frac{ax}{x+b}$

$y' = \frac{a(x+b)-ax(1)}{(x+b)^2}$

$= \frac{ax+ab-ax}{(x+b)^2} = \frac{ab}{(x+b)^2}$

$y' =$ $\frac{ab}{(x+b)^2}$

2. (10 points) A function is given by the following table:

x	0.0	5.0	10.0	15.0	20.0
$f(x)$	10.0	13.0	21.0	34.0	52.0

(a) Make a table for $f'(x)$.

x	2.5	7.5	12.5	17.5
$f'(x)$	.6	1.6	2.6	3.6

① $\frac{\Delta f}{\Delta x} = \frac{13-10}{5-0} = \frac{3}{5} = .6$

② $\frac{\Delta f}{\Delta x} = \frac{21-13}{10-5} = \frac{8}{5} = 1.6$

③ $\frac{\Delta f}{\Delta x} = \frac{34-21}{15-10} = \frac{13}{5} = 2.6$

④ $\frac{\Delta f}{\Delta x} = \frac{52-34}{20-15} = \frac{18}{5} = 3.6$

(b) Make a table for $f''(x)$.

x	5	10	15
$f''(x)$	.2	.2	.2

$\boxed{1}$ $\frac{\Delta f'}{\Delta x} = \frac{1.6-.6}{7.5-2.5} = \frac{1}{5} = .2$

$\boxed{2}$ $\frac{\Delta f'}{\Delta x} = \frac{2.6-1.6}{12.5-7.5} = \frac{1}{5} = .2$

$\boxed{3}$ $\frac{\Delta f'}{\Delta x} = \frac{3.6-2.6}{17.5-12.5} = \frac{1}{5} = .2$

3. (15 points) A small company make coffee mugs with the USC logo on then to sell at local stores. Assume the cost of making 100 mugs is

$$C(100) = \$400.00$$

and the marginal cost for the 100th mug is

$$MC(100) = \$2.50$$

(a) Estimate the total cost making 104 mugs. $C(104) \approx$ \$410.00

$C(104) \approx C(100) + MC(100)(4)$
$= 400 + (2.50)(4)$
$= 410$

(b) If $MR(100) = \$2.75$ should the company ***increase*** or ***decrease*** their production (circle one). Write a sentence or two explaining why.

Producing the 101st mug costs \$2.50 and brings in \$2.75 for a net profit of \$.25 > 0 so the company should increase production

4. Use your calculator to find the maximum and maximizer for the function $f(x) = 10xe^{-.4x}$ on the interval $0 \leq x \leq 10$.

$Y1 = 10Xe^{\wedge}(-.4X)$

$X_{min} = 0$, $X_{max} = 10$

Maximizer 2.500

Minimizer 0 (clear from graph)

2^{nd} calc 4: maximum

left bound = 0

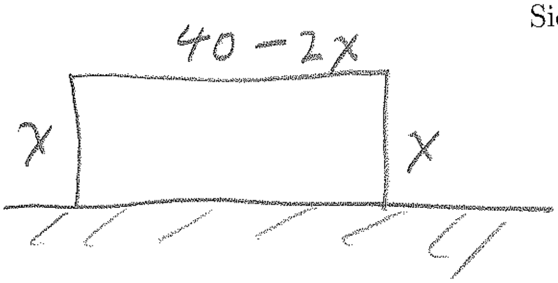
right bound = 5

$X = 2.5000$ $Y = 9.19645$

5. You have 40 feet of fencing to enclose a rectangular area up against a long straight wall. What is the maximum area you can enclose?

Maximum area is 200

Side lengths of the rectangle. $x = 10$, $40 - 2x = 20$



Let the sides of the fence have lengths x and $40-2x$ as shown. Then the area is

$A = x(40-2x)$

$= 40x - 2x^2$

$\frac{dA}{dx} = 40 - 4x = 0$

$\Rightarrow x = 10$

so $x = 10$ is critical point.

$+ + + \mid_{10} - - -$ $\frac{dA}{dx}$

so $x = 10$ is a global maximizer.

The maximum area is

$A = 10(40 - 2(10)) = 200$