Mathematics 122 Test #2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) Let $c > 0$ be a constant and define a function $f(t)$ on $0 \le t \le c$ by

$$f(t) = t^2(c - t).$$

Find the global maximum and maximizer of $f(t)$.

$f(t) = ct^2 - t^3$

$f'(t) = 2ct - 3t^2$

$= t(2c - 3t) = 0$

implies $t = 0$ or $t = \frac{2}{3}c$

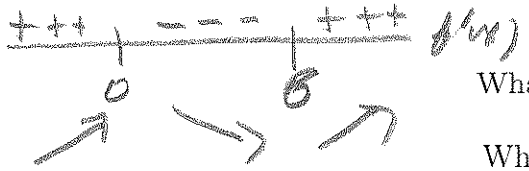
Global maximizer $\frac{2}{3}c$

Global maximum $\frac{4c^3}{27}$

Thus $t = \frac{2}{3}c$ is a global maximizer

The maximum is $f(\frac{2}{3}c) = (\frac{2}{3}c)^2(c - \frac{2}{3}c) = \frac{4c^3}{27}$

2. (10 points) Let $y = f(x)$ be a function such that $f'(x) > 0$ for $x < 0$ and $6 < x$, and $f'(x) < 0$ for $0 < x < 6$.



What are the critical points of $f(x)$? 0, 6

What are the local maximizer(s) of $f(x)$? 0

What are the local minimizer(s) of $f(x)$? 6

3. (10 points) On the interval $0 \le x \le 3$ let

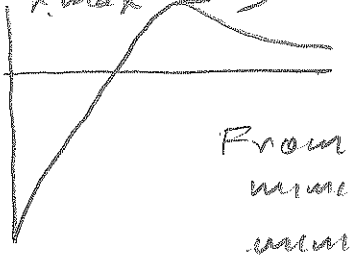
$$f(x) = \frac{x - 1}{5 + x^4}.$$

Find

Y1 = (X−1)/(5+X^4)

Xmin = 0

Xmax = 3



From graph minimum occurs at 0

minimum = f(0) = −.2

use 2nd calc 4: maximum to find max

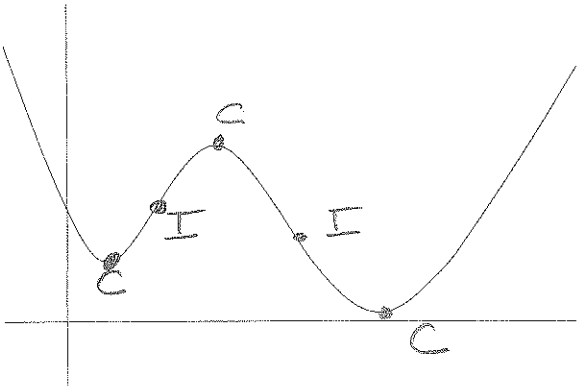
The global maximum of $f(x)$ .05244

The global minimum of $f(x)$ −.2

The global maximizer of $f(x)$ 0

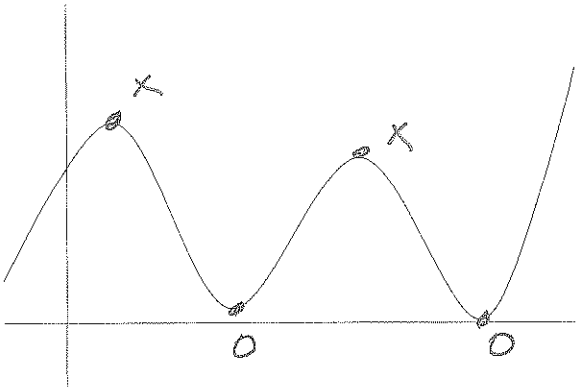
The global minimizer of $f(x)$ 1.6829

4. (10 points) (a) On the follow graph label the critical point with a **C** and all the inflection points with an **I**



C where $f'=0$
I where f'' changes sign

(b) On the follow graph label the local maximums with a **X** and the local minimums with a **O**.



5. (5 points) Find the inflection point (give both x and y values) on the graph of y of $y = 2x^3 + 18x^2 - 4x + 2$.

$y = f(x) = 2x^3 + 18x^2 - 4x + 2$

$f'(x) = 6x^2 + 36x - 4$

$f''(x) = 12x + 36$

$f''(x) = 12x + 36 = 0$

$x = -\frac{36}{12} = -3$ which is inflection point as $f''(x)$ changes sign

y value $= f(-3) = 2(-3)^3 + 18(-3)^2 - 4(-3) + 2$

$= 122$

Inflection point is $(-3, 122)$

6. (10 points) The following table gives the speed, v, in feet/sec of a car t seconds after its brakes are applied.

t	0.0	0.5	1.0	1.5	2.0
v	45	35	20	10	0

$\Delta t = .5$

Compute the following:

$.5(45+35+20+10)$

$.5(35+20+10+0)$

$\frac{55+32.5}{2}$

Upper bound for the distance traveled. 55

Lower bound for the distance traveled. 32.5

Best guess of the distance traveled. 43.75

7. (15 points) Under ideal conditions a loquat tree will produce 120 lbs of fruit . I am going to plant loquat trees in my back yard. The yard is small enough that the trees are a bit overcrowded and each new tree in the yard reduces the yield of all the trees by 20 lbs of fruit. (This even with just one tree it will only product $120 - 20 = 100$ lbs of fruit.)

(a) If I plant x trees, what is the total yield, y, of fruit in lbs.

$y =$ (# of trees)(yield per tree) $y =$ $x(120-20x)$

$= x(120-20x)$

(b) How many trees should I plant to maximize the yield?

$y = 120x - 20x^2$

$y' = 120 - 40x = 0$

$x = \frac{120}{40} = 3$

(or use calculator)

Number of trees is x

+ + + 3 − − − 40

so $x = 3$ is max

(c) What is the maximum yield?

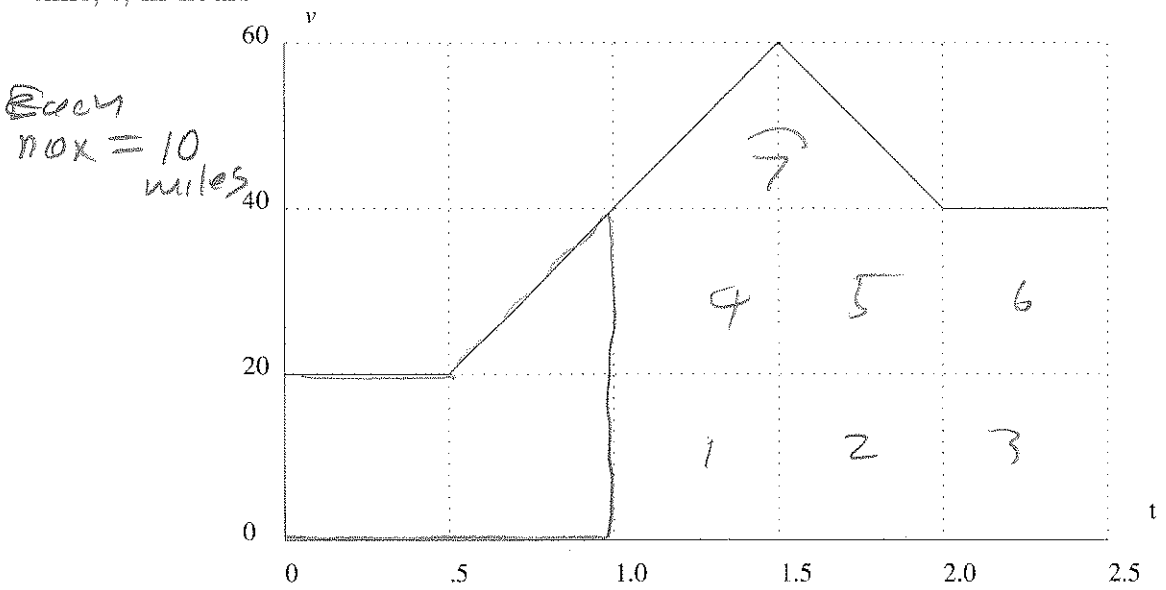
Maximum yield is 180

$y = 3(120 - 20(3))$

$= 3(120 - 60)$

$= 3(60) = 180$

8. (10 points) The following figure shows the velocity, v, in miles/hour of a car as a function of time, t, in hours



Each box = 10 miles

(a) How far did the car travel in the first hour of the trip?

2.5 boxes $= 2.5(10) = 25$

Distance traveled 25 miles

(b) How far did the car travel between $t = 1.0$ and $t = 2.5$?

7 boxes $= 7(10)$ miles

Distance traveled 70 miles

9. (10 points) Use your calculator to compute the following:

fnInt(X^2,X,0,3)

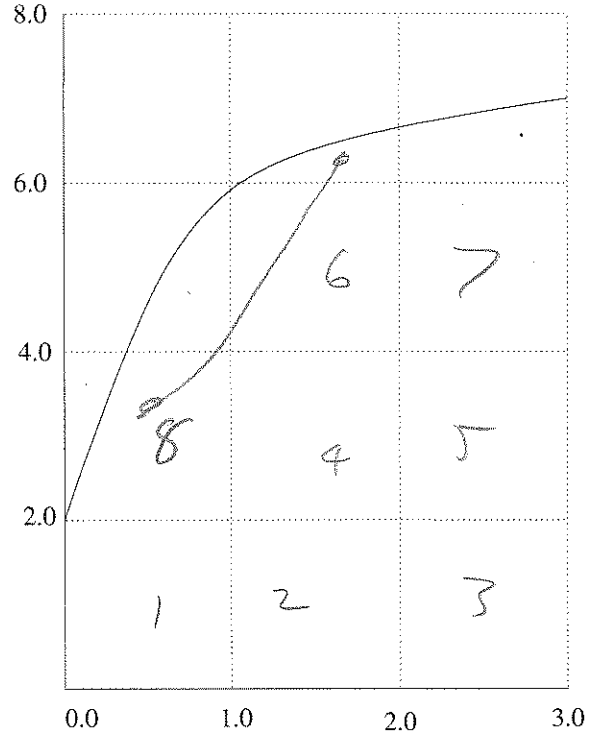
$\int_0^3 x^2\,dx =$ 9

fnInt((1+X^2)/(2^X+1),X,-1,5)

$\int_{-1}^5 \frac{1+x^2}{2^x+1}\,dx =$ 5.2215

10. (5 points) Using the following graph estimate $\int_0^3 f(x)\,dx$

$\int_0^3 f(x)\,dx \approx$ 17.5 (or other answers close to this are ok.



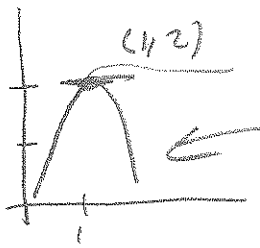
Each box has area = 2

I count ≈ 8.75 boxes

so $\int_0^3$ 8 boxes ≈ (8.75)(2) = 17.5

11. (5 points) Let $f(x)$ be a function with $f(1) = 2$, $f'(1) = 0$ and $f''(x) < 0$ for all x.

(a) Draw a graph that fits this information.



horizontal tangent as $f'(1) = 0$

concave down as $f'' < 0$

(b) The point $x = 1$ is either a ***global maximizer*** or a ***global minimizer*** (circle one) and give brief explanation of how you figured this out (you can refer to your picture in part (a)).

since the graph is concave down the highest point is where $f' = 0$.