Mathematics 122

Quiz # 15 Name: Key

You must show your work to get full credit.

1. Let a be a constant and

$$f(x) = x^2(a - x)$$

for $0 \le x \le a$.

$f(x) = x^2a - x^3$

$f'(x) = 2ax - 3x^2$

$= x(2a - 3x)$

so the critical points are $x = 0, x = \frac{2a}{3}$

endpoints $x = 0, a$

$f(0) = f(a) = 0$

$f(\frac{2a}{3}) = (\frac{2a}{3})^2(a - \frac{2a}{3}) = \frac{4a^2}{9}(\frac{a}{3}) = \frac{4a^3}{27}$

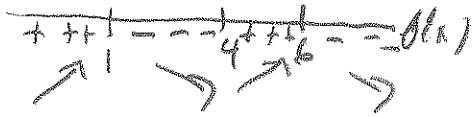
Maximizer $\frac{2a}{3}$

Maximum $\frac{4a^3}{27}$

Minimizers $0, a$

Minimum 0

2. Let a function satisfy $f'(x) > 0$ for $x < 1$ and $4 < x < 6$, and $f'(x) < 0$ for $1 < x < 4$ and $6 < x$.

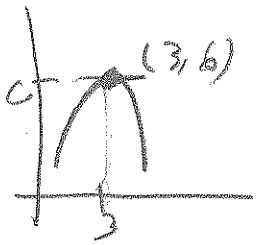


What are the critical points of $f(x)$? 1, 4, 6

What are the local maximizers of $f(x)$? 1, 6

What are the local minimizers of $f(x)$? 4

3. Draw a graph of a function with $f(3) = 6$, $f'(3) = 0$, and $f''(x) < 0$ for all x.



concave down

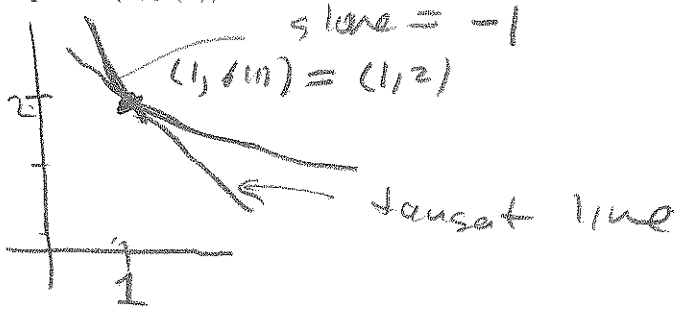
What can you say about the point $x = 3$? It is a global maximizer.

What can you say about the value $f(3) = 6$? It is a global maximum

concave up

4. (15 points) Let $f(x)$ be a function with $f(1) = 2$, $f'(1) = -1$, and $f''(x) > 0$.

(a) Draw a graph that fits this data, label the point $(1, f(1))$ and include the graph of the tangent line at the point $(1, f(1))$.



(b) What is the equation of the tangent line to $y = f(x)$ and the point where $x = 1$?

$y = y_0 + m(x - x_0)$
$= 2 + (-1)(x-1)$
$= 2 - x + 1$
$= 3 - x$

Equation of tangent line is $y = 3 - x$

or $y = 2 - (x-1)$

(c) Estimate $f(1.04)$ $\approx 2 - (1.04 - 1)$
$= 2 - .04$
$= 1.96$

$f(1.04) \approx$ 1.96

5. (10 points) Find the equation of the tangent line to $y = 10 - x^2$ at the point where $x = 2$. Show all your work.

Point slope form of line is
$y = y_0 + m(x - x_0)$

Here $x_0 = 2$, $f'(x) = (10 - x^2)' = 0 - 2x = -2x$

$m = f'(x_0) = f'(2) = -2(2) = -4$

$y_0 = f(x_0) = 10 - (2)^2 = 6$

$y = 6 - 4(x - 2)$
$= 6 - 4x + 8$
$= 14 - 4x$

Equation of tangent line is $y = 6 - 4(x-2)$
$= -4x + 14$