Mathematics 122

Quiz # 21 Name: Key

Y ou must show your work to get full credit.

The ***Fundamental Theorem of Calculus*** says that if $f(x)$ is the derivative of $F(x)$, that is $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

For example if $f(x) = 12x^3$, then if $F(x) = 3x^4$ we have $F'(x) = 12x^3 = f(x)$ and therefore

$$\int_1^3 12x^3\,dx = 3x^4\Big|_1^3 = 3(3)^4 - 3(1)^4 = 240.$$

For anther example if c is a constant and $f(x) = cx^3$, then for $F(x) = \frac{1}{4}cx^4$ we have $F'(x) = f(x)$. Thus

$$\int_0^2 cx^3\,dx = \frac{1}{4}cx^4\Big|_0^1 = \frac{c(1)^2 - c(0)^4}{4} = \frac{c}{4}.$$

Use the Fundamental Theorem of Calculus to find the exact values of the following

1. $\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} - \frac{0}{3} = \frac{1}{3}$ $\int_0^1 x^2\,dx =$ $1/3$

2. $\int_{-1}^1 e^x\,dx = e^x\Big|_{-1}^1 = e^1 - e^{-1}$ $\int_{-1}^1 e^x\,dx =$ $e - \frac{1}{e}$

3. $\int_a^b 3x^2\,dx = x^3\Big|_a^b$
$= b^3 - a^3$ $\int_a^b 3x^2\,dx =$ $b^3 - a^3$