**Mathematics 172**

**Quiz 17** **Name:** Key

***You must show your work to get full credit.***

Consider the matrix

$$P = \begin{bmatrix} 0.0 & 3.5 & 10.0 \\ 0.2 & 0.0 & 0.0 \\ 0.0 & 0.25 & 0.0 \end{bmatrix}$$

If we start with

$$\vec{N}_0 = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

**1.** What are $\vec{N}_{30}$ and $\vec{N}_{31}$?

$$\vec{N}_{30} = \begin{bmatrix} 80.87 \\ 14.94 \\ 3.475 \end{bmatrix} \quad \vec{N}_{31} = \begin{bmatrix} 87.23 \\ 16.17 \\ 3.75 \end{bmatrix}$$

**2.** What is the proportion in each stage of $\vec{N}_{30}$ Proportion in Stage 1 .8141

Proportion in Stage 2 .1509

Proportion in Stage 3 .0350

**3.** To check to see if $\vec{N}_{30}$ has reached the stable distribution we need to check if there is a $\lambda$ such that

$$\vec{N}_{31} = \lambda \vec{N}_{30}.$$

holds to high accuracy. To do this note this vector equation leads to three equations for $\lambda$ and thus three value for $\lambda$.

Equation 1 is $80.87\lambda = 87.23$ Gives value of $\lambda$ as 1.079

Equation 2 is $14.94\lambda = 16.17$ Gives value of $\lambda$ as 1.079

Equation 3 is $3.475\lambda = 3.75$ Gives value of $\lambda$ as 1.079

Is this good enough accuracy?

**4.** As in discrete exponential growth the per capita growth rate is given by $r = \lambda - 1$.

The per capita growth rate is $r =$ .079