Mathematics 172

Quiz 7 Name: Key

You must show your work to get full credit.

1. Let a population be modeled by

$$N_{t+1} = N_t + 1.3N_t\left(1 - \frac{N_t}{100}\right)$$

(a) What are the equilibrium points? 0, 100

(b) Which equilibrium pts are stable? 100

(c) Which equilibrium points are unstable? 0

(a) solve $N_{t+1} = N_t$, i.e. $\cancel{N_t} + 1.3 N_t\left(1-\frac{N_t}{100}\right) = \cancel{N_t}$

$1.3 N_t\left(1-\frac{N_t}{100}\right) = 0 \Rightarrow N_t = 0, 100$

(b),(c) $N_{t+1} = f(N_t)$ where $f(N) = N + 1.3N\left(1-\frac{N}{100}\right) = N + 1.3N - \frac{1.3N^2}{100}$

$f'(N) = 1 + 1.3 - \frac{2(1.3)N}{100}$

$f'(0) = 1 + 1.3 = 2.3 > 1$ so 0 unstable

$f'(100) = 1 + 1.3 - \frac{2(1.3)100}{100} = 1 + 1.3 - 2(1.3) = -.3$ so stable

2. For a population modeled by

$$\Delta P = .3P\left(1 - \frac{P}{75}\right)$$

(a) What are the equilibrium points? 0, 75

(b) Which equilibrium are stable? 75

(c) Which equilibrium points are unstable? 0

(d) For this model do you expect the population to settle down so some fixed size? If so what value? Give a sentence or two (and maybe a picture) to justify your answer.

(a) solve $\Delta P = .3P\left(1-\frac{P}{75}\right) = 0$ so $P = 0, 75$

(b),(c) $P_{t+1} = P_t + \Delta P_t = P_t + .3P_t\left(1-\frac{P_t}{75}\right) = f(P_t)$

where $f(P) = P + .3P - \frac{.3P^2}{75}$

$f'(P) = 1 + .3 - \frac{2(.3)(P)}{75}$

$f'(0) = 1 + .3 - 0 = 1.3 > 1$ so unstable

$f'(75) = 1 + .3 - \frac{2(.3)(75)}{75} = .7 < 1$ so stable

(d) We can expect the population size to settle down to 75, the stable equilibrium point.