Mathematics 172

Quiz 9 Name: Key

You must show your work to get full credit.

1. If we model a population with

$$P' = .15P \qquad P(0) = 200$$

how long does it take for the population to double in size?

Doubling time is 4.621

The solutio to $p' = rP$ is $P(t) = P_0 e^{rt}$

In our case this is $P(t) = 200e^{.15t}$

we wish to solve $P(t) = 2P(0)$ for t

$200e^{.15t} = 2(200)$

$e^{.15t} = 2$

$.15t = \ln(2)$

$t = \ln(2)/.15 = 4.621$

2. We model population growth by the modified logistic equation

$$\frac{dN}{dt} = .1N\left(1 - \left(\frac{N}{500}\right)^2\right)$$

(a) What are the rest points for this equation? Rest points are 0, 500

Solve $\frac{dN}{dt} = .1N\left(1-\left(\frac{N}{500}\right)^2\right) = 0$

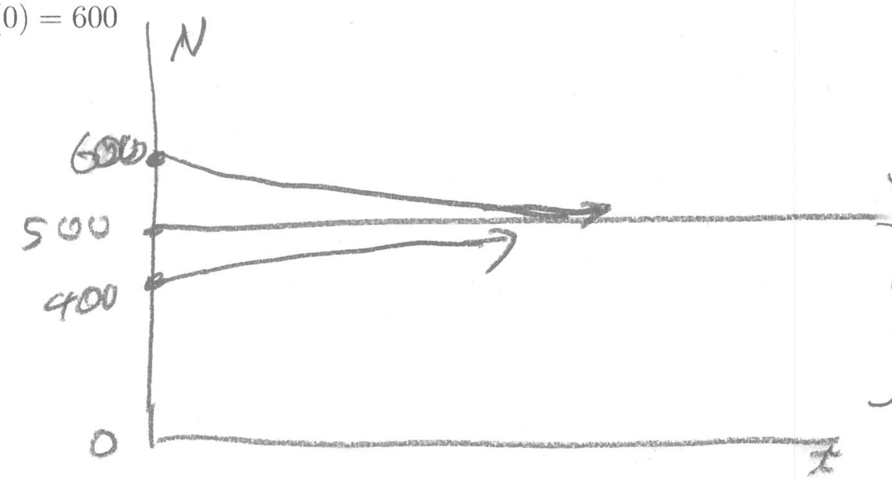
$N = 0, \quad 1 - \left(\frac{N}{500}\right)^2 = 0$

$\left(\frac{N}{500}\right)^2 = +1$

$\frac{N}{500} = \sqrt{1} = 1$

$N = 500$

(b) Draw a graph showing the rest solutions along with the solutions with $N(0) = 400$ and $N(0) = 600$



$\frac{dN}{dt} = .1N\left(1-\left(\frac{N}{500}\right)^2\right) < 0$ so $N \searrow$

$\frac{dN}{dt} = .1N\left(1-\left(\frac{N}{500}\right)^2\right) > 0$ so $N \nearrow$

(c) If $N(0) = 400$ estimate $N(100)$. $N(100) \approx$ 500