Mathematics 172 Homework

We have seen in class that if two organisms have the same shape but the second is λ times the length of the first, then

$$\text{Surface area of second} = \lambda^2 \cdot (\text{Surface area of first})$$

$$\text{Volume of second} = \lambda^3 \cdot (\text{Volume of first}).$$

And if the density of the two is the same (say they are of the same species) then the weight is propositional to the volume and thus

$$\text{Weight of the second} = \lambda^3(\text{Weight of the first}).$$

Here is an example of how this can be used. A great white shark is between four and five feet when born and are very close in shap to the adults. Assume that a great white shark pup is 5 feet long and weighs 77 pounds. We use this to make a guess at the weight of the shark when it is 12 feet long. Here we have scaled the length by

$$\lambda = \frac{12}{5} = 2.4$$

Therefore we expect the weight of the 12 foot version to be

$$W = \lambda^3(\text{Weight of 5 foot version}) = (2.4)^3 \cdot 77 = 1{,}64.44\text{lbs}.$$

Here are some problems for you to try.

1. A large male domestic cat is 9.5 inches in height and weighs 10 lbs. The height of a large male Siberian tiger is 43 inches. What would be the weight of a male domestic cat if its was scaled up to have a height of 43 inches? *Solution:* The weight of the rescaled domestic would be 927.331 lbs. For comparison the weight of a large Siberian tiger is 661 lbs.

2. The largest snake in the fossil record is the titanoboa, which lived about 60 million years ago in what it now Columbia South America. The largest of these were about 42 feet long. Currently the largest boa in the world is the anaconda. The largest measured to date was 17.09 feet long and weighed 215 pounds. Assuming that the titanoboa had the same basic body plan as an anaconda, estimate the weight of a 42 foot titanoboa. *Solution:* The estimated weight would be 3191.245 lbs. (According to Wikipedia this estimate is too hight, meaning that the titanoboa would have had a slimmer build than the anaconda.)

3. The longleaf pine will grow up to 120 feet tall. If a 10 foot tall longleaf pine weighs 100 lbs, estimate the weight of a longleaf that is 120 feet tall. *Solution:* The estimate is 172,800 pounds, which is 86.4 tons.